Downside Risk Optimization vs Mean-Variance Optimization

Author: Andrea Rigamonti[1]
Discussant: Wolfgang Schadner[2]

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[1]Free University of Bozen, Faculty of Economics and Management
[2]University of St.Gallen, Swiss Institute of Banking and Finance

Short Summary

- Investors see not all variability in returns as risk, only the one below a certain level (e.g. 0 or r_F)
  $\Longrightarrow$ if distribution is unsymmetrical, MV-optim. does not match preferences
- MV common practice, but investors should (theoretically) care more about the downside
- discusses and tests a framework of mean-semivariance optimization
- validation upon expected shortfall (instead of semi-variance)

Findings

- semi-cov. matrix is endogenous
  $\implies$ resource intensive $\implies$ less attractive for investors

- out-of-sample: MV approach outperforms downside one
  (also when switching from semi-variance to expected shortfall)
  $\implies$ due to parameter uncertainty

- theoretically, Sortino ratio should be preferred above Sharpe ratio, empirically not

- estimation of downside risk is too in-precise

- Negative return correlation may cause bias when estimating semi-cov. matrix

Discussion and Suggestions

1. Investors can also short sell stocks, then they want to maximize the "downside risk"
2. Investors have S-shaped preferences, what about a mixture of upward & downward risk measure with higher weight on the latter
3. Increasing return frequency, does it reduce parameter uncertainty?

Discussion and Suggestions - Cont.

4 Maybe drawing-with-replacement (in time) is not the best technique due to heteroskedasticity, what about d.w.r. at the cross-section?
MSV Optimization may have it's time, e.g. different performance during crises

5 Two assets only - diversification effects?

6 Might be interesting: there's a forthcoming JF article showing evidence for the (ex-ante) downside risk anomaly.
(Schneider P., Wagner Z., Zechner J. (2019): Low Risk Anomalies?, Journal of Finance)

Fear and Laughing of the Market

trending pessimism, fragile optimism

Wolfgang Schadner

Swiss Institute of Banking and Finance
University of St.Gallen

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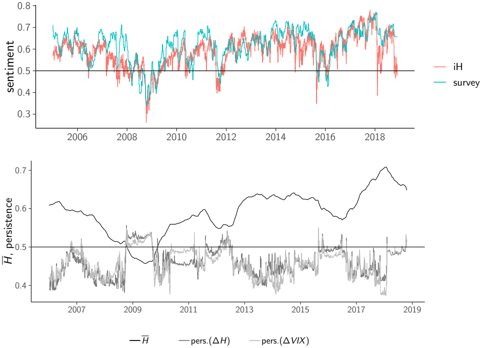
Structure

On an equity market level...



1. Introduce a new (directional) sentiment measure

2. Show some empirical patterns of it

Motivation - Research Question

research on equity option's implied volatilities

- typically risk-neutral distribution
- risk-neutral auto-correlation?

fractal option pricing

$\downarrow$

implied return persistence (H)

$\downarrow$

risk-neutral/expected momentum

$\downarrow$

market sentiment

Relevant Literature

- (Risk-Neutral) Asset Pricing
  e.g. Cochrane [2005], Björk [2009]

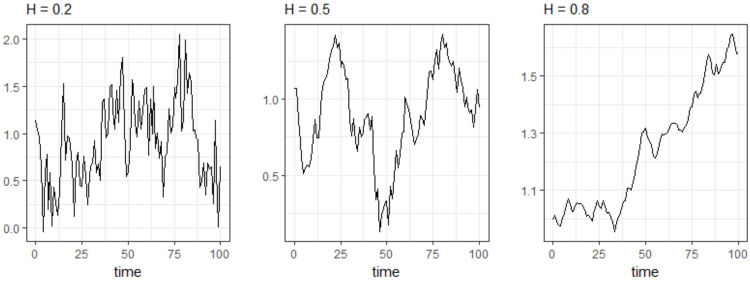
- Fractal BM and its Implications
  e.g. Black & Scholes [1973], Hurst[1956], Mandelbrot & Van Ness [1968], Hu & Oksendal [2003]

- Market Sentiment
  e.g. Whaley [2000], Baker & Wurgler [2007], Caporale et al. [2018]

- extends my previous work (Schadner [2019])

Some words on fractal Brownian motion

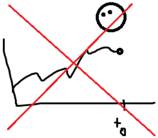
$H \in (0, 1)$:



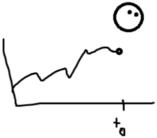
$\Downarrow$	$\Downarrow$	$\Downarrow$
neg. auto-cor.	no auto-cor.	pos. auto-cor.
anti-persistent	classic BM	trending

Similar to VIX

- ~~ex-post~~ → ex-ante



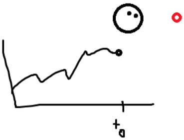
experience



expectation

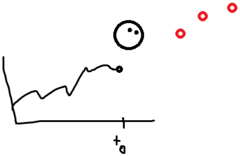
- Implied vola, comparison:

CBOE VIX:



1 month, σ

Fractal Approach:



term structure, H

Hold it!

The idea:

If

$$\text{auto-correlation} = \text{momentum}$$

then

$$\text{expected auto-correlation} = \text{expected momentum}$$

Why implied H = sentiment

Start with

$$1 = \mathbb{E}[m \cdot R]$$

and

$$dS_t = \mu S_t \, dt + \sigma S_t \, dB_t^H$$

so with CRRA, $\mathbb{Q}$-dynamics and some magic we get

$$\bar{r} = (\mu - r_f) = \gamma\sigma^2 \underbrace{\tau^{2(H-0.5)}}_{\substack{=1 \text{ under} \\ \text{classic BM}}} \qquad \text{...excess return}$$

$$\implies \bar{r} = f(\tau):$$

$$\frac{\partial \bar{r}}{\partial \tau} = (H - 0.5) \cdot \left(2\gamma\sigma^2\tau^{2(H-1)}\right)$$

In easy words

H

$> 0.5 \implies \frac{\partial \bar{r}}{\partial \tau} > 0 \implies$ $+$ R.N. Mom.
MRP expect. to increase

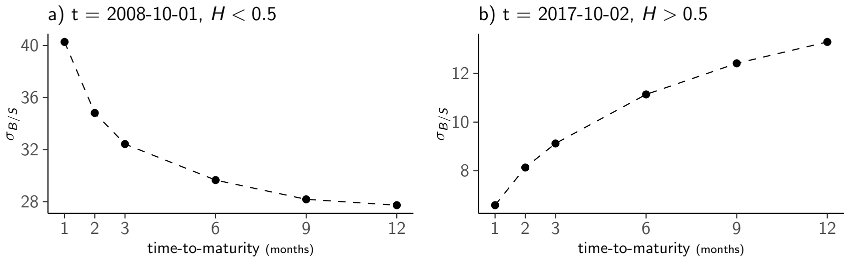
$= 0.5 \implies \frac{\partial \bar{r}}{\partial \tau} = 0 \implies$ no R.N. Mom.
MRP expect. to remain

$< 0.5 \implies \frac{\partial \bar{r}}{\partial \tau} < 0 \implies$ $-$ R.N. Mom.
MRP expect. to decrease

Estimating implied H

OLS-Regression[1]:

$$\underbrace{\ln(\sigma_{B/S})}_{\hat{y}} = \underbrace{\ln(\sigma_f)}_{\hat{\alpha}} + \underbrace{(H - 0.5)}_{\hat{\beta}} \cdot \underbrace{\ln(\tau)}_{x}$$



$\Longrightarrow$ H indicates slope of the implied vola. term structure

[1] from Hu and Oksendal [2003]

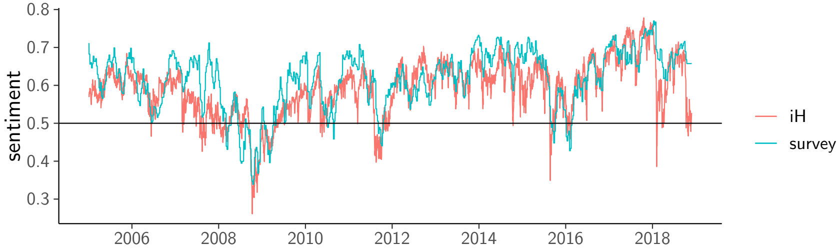
Worth to mention

- Supportive Alternative
  VIX = market fear; what if $\text{VIX}_{1\text{m}} > \text{VIX}_{12\text{m}}$?

- Variance risk-premium
  if
  $$\sigma_{implied} = \mathbb{E}[\sigma] + VRP$$
  then
  $$\text{sgn}\left(\frac{\partial \bar{r}}{\partial \tau}\right) \perp VRP$$
  so
  $$implied\ H = expected\ H$$

Note: literature often defines VRP this way.

Example - S&P 500



iH vs. surveys:

- higher frequency, directly from traded data, complete picture of market

iH vs. VIX:

- directional interpretation (*bullish*/*bearish*), theoretical arguments

Empirical Research Set-Up

- Observation horizon 2007 to April 2019
- Data from Datastream and Bloomberg, ATM options
- H estimated daily for 8 different regions
  - U.S.
  - U.K.
  - France
  - Germany
  - Japan
  - Netherlands
  - Switzerland
  - Euro-Zone
- R^2 between 84% to 99%[2]

[2] 1st and 3rd quartile

Verifying H = sentiment

United States (US)

.VIX	.1	.2	.3	.4	.5	.6	.7	.8	.9
-0.84	0.78	0.71	0.59	0.57	0.51	0.44	0.35	0.22	0.15
***	***	***	***	***	***	***	***	***	*

Euro Zone (EUR)

.VX	.1	.2	.3	.4	.5	.6	.7	.8	.9	.10
-0.81	0.53	0.52	0.46	0.40	0.39	0.37	0.37	0.36	0.35	0.26
***	***	***	***	***	***	***	***	***	***	***

France (FRA)

.VX	.1	.2	.3	.4	.5	.6	.7
-0.84	0.51	0.35	0.34	0.33	0.21	0.17	0.01
***	***	***	***	***	**	*	

Germany (GER)

.VX	.1	.2	.3	.4	.5	.6	.7	.8	.9
-0.83	0.47	0.40	0.38	0.36	0.34	0.31	0.30	0.15	-0.04
***	***	***	***	***	***	***	***	***	

Japan (JP)

.VX	.1	.2	.3	.4	.5
-0.4	0.30	0.19	0.10	-0.3	0.00 0
***	***	***	***	*	

Netherlands (NED)

.VX	.1	.2	.3	.4	.5	.6
-0.8	0.30	0.27	0.20	0.18	0.18	0.17
***	***	***	**	**	**	**

Switzerland (CH)

.VX	.1	.2	.3	.4	.5	.6	.7	.8	.9
-0.77	0.47	0.46	0.42	0.41	0.36	0.28	0.25	0.22	0.12
***	***	***	***	***	***	***	***	**	

United Kingdom (UK)

.VX	.1	.2	.3	.4	.5	.6	.7	.8
-0.82	0.40	0.39	0.31	0.24	0.22	0.20	0.20	0.02
***	***	***	***	***	**	**	**	

Correlations - Cont.

H & VIX and VIX-like

- significantly negative $\rightarrow$ if fear is high, H is low
- -0.43^{***} (Japan) to -0.84^{***} (U.S.)

H & other sentiment

- e.g. consumer confidence, sentiment surveys, fund flows, business confidence, EC economic sentiment, ...
- 71 pairs analyzed $\rightarrow$ 66 are significant

$\Longrightarrow$ implied H indeed measures market sentiment

H's Characteristics

	US	EUR	CH	FRA	GER	JP	NED	UK
				iH				
Mean	0.59	0.52	0.54	0.52	0.54	0.53	0.53	0.56
Std.Dev.	0.08	0.07	0.07	0.08	0.07	0.07	0.07	0.08
				ΔiH				
Mean	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Std.Dev	0.03	0.06	0.06	0.06	0.04	0.07	0.07	0.05
Skewness	-1.07***	-0.17***	-0.20***	-0.49***	-0.29***	-0.82***	-1.12***	-0.55***
Kurtosis	17.85***	9.72***	7.21***	28.25***	7.57***	25.38***	79.58***	12.82***

* $p < 5\%$, ** $p < 1\%$, *** $p < 0.1\%$

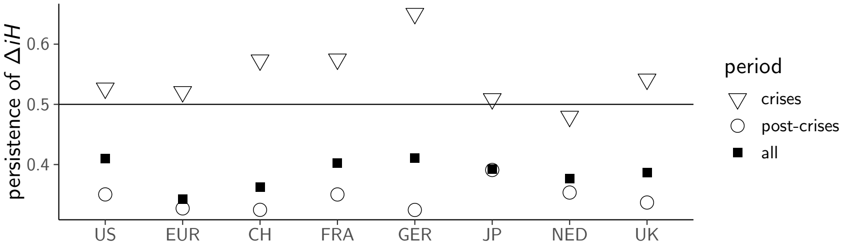
$\Longrightarrow$ fear occurs faster than optimism

Some Thoughts

if market sentiment is skewed in favour of pessimism ...

... than fear has to be more persistent than confidence

Persistence of implied Persistence - static



$\implies$ sentiment is trending during crises, but anti-persistent afterwards[3]

[3] Robustness checks on VIX & VIX-like indices confirm the pattern.

Persistence of implied Persistence - rolling

correlation(*sentiment* & *sentiment persistence*)[4]:

	US	EUR	CH	FRA	GER	JP	NED	UK
$\overline{iH}$	-0.39 ***	-0.09	-0.50 ***	-0.29 **	-0.31 ***	-0.34 ***	-0.36 ***	-0.27 **

$\Longrightarrow$ the better the mood, the less stable it is

$\Longrightarrow$ robust also under VIX indices

[4] 1 year rolling window; $mavg(iH)$ vs. persistence(ΔiH) for same window

Summary

Model:

- fractal BM $\rightarrow$ decomposition of option implied vola. term structure
- implied $H \rightarrow$ market sentiment

From 8 regions and robustness on VIX:

- sentiment is skewed $\rightarrow$ pessimism faster than optimism
- sentiment's persistence varies $\rightarrow$ *bearish* is trending, *bullish* is fragile

Further idea:
... analysis on a firm level

... thank you for listening!