Downside Risk Optimization vs Mean-Variance Optimization

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Short Summary

▶ Investors see not all variability in returns as risk, only the one below a certain level (e.g. 0 or $r_F$)
  $\implies$ if distribution is unsymmetrical, MV-optim. does not match preferences

▶ MV common practice, but investors should (theoretically) care more about the downside

▶ discusses and tests a framework of mean-semivariance optimization

▶ validation upon expected shortfall (instead of semi-variance)
Findings

- semi-cov. matrix is endogenous
  $\implies$ resource intensive $\implies$ less attractive for investors

- out-of-sample: MV approach outperforms downside one
  (also when switching from semi-variance to expected shortfall)
  $\implies$ due to parameter uncertainty

- theoretically, Sortino ratio should be preferred above Sharpe ratio, empirically not

- estimation of downside risk is too in-precise

- Negative return correlation may cause bias when estimating semi-cov. matrix
Discussion and Suggestions

1. Investors can also short sell stocks, then they want to maximize the "downside risk"

2. Investors have S-shaped preferences, what about a mixture of upward & downward risk measure with higher weight on the latter

3. Increasing return frequency, does it reduce parameter uncertainty?
Discussion and Suggestions - Cont.

4 Maybe drawing-with-replacement (in time) is not the best technique due to heteroskedasticity, what about d.w.r. at the cross-section? MSV Optimization may have it’s time, e.g. different performance during crises.

5 Two assets only - diversification effects?

6 Might be interesting: there’s a forthcoming JF article showing evidence for the (ex-ante) downside risk anomaly. (Schneider P., Wagner Z., Zechner J. (2019): Low Risk Anomalies?, Journal of Finance)
Fear and Laughing of the Market
trending pessimism, fragile optimism

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Structure

On an equity market level...

1. Introduce a new (directional) sentiment measure

2. Show some empirical patterns of it
Motivation - Research Question

research on equity option’s implied volatilities

- typically risk-neutral distribution
- risk-neutral auto-correlation?

fractal option pricing
\[ \downarrow \]

implied return persistence \((H)\)
\[ \downarrow \]

risk-neutral/expected momentum
\[ \downarrow \]

market sentiment
Relevant Literature

- (Risk-Neutral) Asset Pricing
  e.g. Cochrane [2005], Björk [2009]

- Fractal BM and its Implications
  e.g. Black & Scholes [1973], Hurst [1956], Mandelbrot & Van Ness [1968], Hu & Oksendal [2003]

- Market Sentiment
  e.g. Whaley [2000], Baker & Wurgler [2007], Caporale et al. [2018]

- extends my previous work (Schadner [2019])
Some words on fractal Brownian motion

\[ H \in (0, 1): \]

- \( H = 0.2 \): neg. auto-cor. anti-persistent
- \( H = 0.5 \): no auto-cor. classic BM
- \( H = 0.8 \): pos. auto-cor. trending
Similar to VIX

- **ex-post → ex-ante**

- Implied vola, comparison:
  - **CBOE VIX:**
  - **Fractal Approach:**
  - 1 month, $\sigma$
  - term structure, $H$
Hold it!

The idea:

If

\[ \text{auto-correlation} = \text{momentum} \]

then

\[ \text{expected auto-correlation} = \text{expected momentum} \]
Why implied $H = $ sentiment

Start with

$$1 = \mathbb{E}[m \cdot R]$$

and

$$dS_t = \mu S_t \, dt + \sigma S_t \, dB^H_t$$

so with CRRA, $\mathbb{Q}$-dynamics and some magic we get

$$\bar{r} = (\mu - r_f) = \gamma \sigma^2 \tau^{2(H-0.5)}$$

...excess return

$$\bar{r} = f(\tau) :$$

$$\frac{\partial \bar{r}}{\partial \tau} = (H - 0.5) \cdot (2\gamma \sigma^2 \tau^{2(H-1)})$$
In easy words

\[
H > 0.5 \implies \frac{\partial \bar{r}}{\partial \tau} > 0 \implies + \text{R.N. Mom.} \\
\text{MRP expect. to increase}
\]

\[
H = 0.5 \implies \frac{\partial \bar{r}}{\partial \tau} = 0 \implies \text{no R.N. Mom.} \\
\text{MRP expect. to remain}
\]

\[
H < 0.5 \implies \frac{\partial \bar{r}}{\partial \tau} < 0 \implies - \text{R.N. Mom.} \\
\text{MRP expect. to decrease}
\]
Estimating implied $H$

OLS-Regression\(^1\):

\[
\ln(\sigma_{B/S}) = \ln(\sigma_f) + (H - 0.5) \cdot \ln(\tau)
\]

---

\[\hat{\beta} \cdot \ln(\tau)\]

\[\hat{\alpha} \cdot \ln(\tau)\]

\[
\hat{y} = \ln(\sigma_f) + (H - 0.5) \cdot \ln(\tau)
\]

\(\Rightarrow H\) indicates slope of the implied vola. term structure

\(^1\)from Hu and Oksendal [2003]
Worth to mention

- Supportive Alternative
  VIX = market fear; what if VIX\(_{1m}\) > VIX\(_{12m}\)?

- Variance risk-premium
  if
  \[
  \sigma_{implied} = \mathbb{E}[\sigma] + VRP
  \]
  then
  \[
  \text{sgn} \left( \frac{\partial \tilde{r}}{\partial \tau} \right) \perp VRP
  \]
  so
  \[
  \text{implied } H = \text{expected } H
  \]

Note: literature often defines VRP this way.
Example - S&P 500

*iH* vs. surveys:
- higher frequency, directly from traded data, complete picture of market

*iH* vs. VIX:
- directional interpretation (*bullish*/*bearish*), theoretical arguments
Empirical Research Set-Up

- Observation horizon 2007 to April 2019

- Data from Datastream and Bloomberg, ATM options

- \( H \) estimated daily for 8 different regions
  - U.S.
  - U.K.
  - France
  - Germany
  - Japan
  - Netherlands
  - Switzerland
  - Euro-Zone

- \( R^2 \) between 84% to 99%\(^2\)

\(^2\)1st and 3rd quartile
Verifying $H = \text{sentiment}$

### United States (US)

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<thead>
<tr>
<th>VX</th>
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### Euro Zone (EUR)

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### Japan (JP)

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### Netherlands (NED)

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### Switzerland (CH)

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### United Kingdom (UK)

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<td>0.20</td>
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Correlations - Cont.

H & VIX and VIX-like

- significantly negative $\rightarrow$ if fear is high, $H$ is low
- $-0.43^*^*$ (Japan) to $-0.84^*^*$ (U.S.)

H & other sentiment

- e.g. consumer confidence, sentiment surveys, fund flows, business confidence, EC economic sentiment, ...
- 71 pairs analyzed $\rightarrow$ 66 are significant

$\implies$ implied $H$ indeed measures market sentiment
## H’s Characteristics

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<th>NED</th>
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<tbody>
<tr>
<td><strong>iH</strong></td>
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<td></td>
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<tr>
<td>Mean</td>
<td>0.59</td>
<td>0.52</td>
<td>0.54</td>
<td>0.52</td>
<td>0.54</td>
<td>0.53</td>
<td>0.53</td>
<td>0.56</td>
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<tr>
<td>Std.Dev.</td>
<td>0.08</td>
<td>0.07</td>
<td>0.07</td>
<td>0.08</td>
<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
<td>0.08</td>
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|       |        |        |        |        |        |        |        |        |
| **ΔiH**|        |        |        |        |        |        |        |        |
| Mean  | 0.00   | 0.00   | 0.00   | 0.00   | 0.00   | 0.00   | 0.00   | 0.00   |
| Std.Dev | 0.03  | 0.06   | 0.06   | 0.06   | 0.04   | 0.07   | 0.07   | 0.05   |
| Skewness | 1.07 | 0.17   | 0.20   | 0.49   | 0.29   | 0.82   | 1.12   | 0.55   |
| Kurtosis | 17.85 | 9.72   | 7.21   | 28.25  | 7.57   | 25.38  | 79.58  | 12.82  |

* p < 5%, ** p < 1%, *** p < 0.1%

⇒ fear occurs faster than optimism
Some Thoughts

if market sentiment is skewed in favour of pessimism ...

... than fear has to be more persistent than confidence
Persistence of implied Persistence - static

sentiment is trending during crises, but anti-persistent afterwards\(^3\)

\(^3\)Robustness checks on VIX & VIX-like indices confirm the pattern.
Persistence of implied Persistence - rolling

correlation\((\text{sentiment} \& \text{sentiment persistence})^4\):

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<tr>
<td>(iH)</td>
<td>-0.39</td>
<td>-0.09</td>
<td>-0.50</td>
<td>-0.29</td>
<td>-0.31</td>
<td>-0.34</td>
<td>-0.36</td>
<td>-0.27</td>
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\(\Rightarrow\) the better the mood, the less stable it is

\(\Rightarrow\) robust also under VIX indices

\(^4\) 1 year rolling window; \(mavg(iH)\) vs. persistence\((\Delta iH)\) for same window
Summary

Model:
- fractal BM → decomposition of option implied vola. term structure
- implied $H \rightarrow$ market sentiment

From 8 regions and robustness on VIX:
- sentiment is skewed $\rightarrow$ pessimism faster than optimism
- sentiment’s persistence varies $\rightarrow$ bearish is trending, bullish is fragile

Further idea:
... analysis on a firm level

... thank you for listening!