University of Applied Sciences BFI Vienna

A dynamic MoM copula model approach for market risk estimates

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1. Summary & Introduction

- We analyse models to estimate one-day 1%-return-quantiles (→ 99% VaR risk measure)
- We propose copula-based models using rolling windows of 250 trading days for calibration:
  - Meta-Gaussian models (Gaussian copula)
  - Meta-Student *t* models (Student *t* copula)
- Copula calibration:
  - Pseudo-log-likelihood method (MLE)
  - Method-of-moments (MoM)
- EGARCH-volatility adjusted returns
- Data: daily returns of 21 financial assets from 5 January 1990 to 26 November 2024
- Unconditional and conditional hit tests
- **Best performance: meta-Student *t* model calibrated with the method-of-moments (MoM)**

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- 2.1. Volatility-adjusted returns
- 2.2. Variance-covariance models
- 2.3. Historical simulation models
- 2.4. Copula-based models

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2. Models

- We use nine models to estimate the 1%-quantile for daily portfolio returns

Table 1: Summary of Value-at-Risk models used

Model	Model-type	Return Data
VC_unadj	Variance-covariance model	(unadjusted) log-returns
VC_EWMA	EWMA-variance-covariance model	(unadjusted) log-returns
VC_adj	Variance-covariance model	volatility-adjusted log-returns
HS_unadj	Historical simulation	(unadjusted) log-returns
HS_adj	Historical simulation	volatility-adjusted log-returns
Gauss_MLE	Meta-Gaussian (MLE)	volatility-adjusted log-returns
t_MLE	Meta-Student *t* (MLE)	volatility-adjusted log-returns
Gauss_MoM	Meta-Gaussian (method-of-moments)	volatility-adjusted log-returns
t_MoM	Meta-Student *t* (method-of-moments)	volatility-adjusted log-returns

Notes: Maximum-likelihood estimation (MLE) based on pseudo-observations for copula estimation in *models Gauss_MLE* and *t_MLE*.

2.1. Volatility-adjusted returns

- As an alternative to unadjusted historical log-returns, i.e. $r_t = \ln \frac{S_t}{S_{t-1}} = \ln S_t - \ln S_{t-1}$, these returns are adjusted to account for a changing level of volatility (Duffie and Pan, 1997, Hull and White, 1998, Alexander, 2008).
- We use an **EGARCH (1, 1)** model to estimate time series of volatility estimates (Nelson, 1991).
  In some few cases (223 cases, i.e 0.13% of the cases) the GARCH estimation did not converge → EWMA volatilities
- For every single trading day in our analysis, we calculate time series of volatility-adjusted returns, using a rolling window of 500 trading days. For trading days 251 to 500 the volatility-adjusted returns $\{\tilde{r}_t\}_{t=251}^{T=500}$ are calculated based on unadjusted historical log-returns $\{r_t\}_{t=251}^{T=500}$ and the EGARCH-volatilities $\{\hat{\sigma}_t\}_{t=251}^{T=500}$ as

$$\tilde{r}_t = r_t \cdot \frac{\hat{\sigma}_{500}}{\hat{\sigma}_t} \;\forall\; t \in \{251, \dots, 500\}$$

2.2. Variance-covariance models

- This approach assumes that asset returns are **multivariate normally distributed.**
- Here, the 1% return quantile for the next trading day, $Q_{0.01,t+1}$, is estimated as

$$Q_{0.01,t+1} = \mu_{pf,t} + \sigma_{pf,t} \cdot \Phi^{-1} \cdot (0.01)$$

where for our 21-dimensional portfolio with weights $w_i = \frac{1}{21} \forall i \in (1, \ldots, 21)$,

$\mu_{pf,t} = \frac{1}{250} \sum_{k=t-249}^{t} \mathbf{w}' \cdot \mathbf{r}_k$ is the expected one-day portfolio return and

$\sigma_{pf,t} = \sqrt{\mathbf{w}' \cdot \mathbf{\Sigma}_\mathrm{t} \cdot \mathbf{w}}$ is the estimate for the one-day portfolio volatility (standard deviation of returns) with covariance matrix $\mathbf{\Sigma}_\mathrm{t}$ based on the information until trading day t and

$\Phi^{-1} \approx -2.326348$ is the 1%-quantile of the standard normal distribution.

2.3. Historical simulation models

- Historical simulation models do not make any assumption about the distribution of the portfolio returns.
  No parameters such as covariances or correlations are **estimated**.
- Firstly, historical portfolio returns are calculated for trading days $t-249$ to t, using the portfolio weights of day t (in our case all weights are equal):

$$\boldsymbol{r}_{pf,t} = \mathbf{R}_t \cdot \mathbf{w}$$

  where $\boldsymbol{r}_{pf,t}$ is a column vector containing 250 historical portfolio returns, $\mathbf{R}_t$ is a 250 x 21 matrix containing the most recent 250 returns of the 21 financial portfolios and $\mathbf{w}$ is the weight-vector.
- The 1%-quantile estimate for trading day $t+1$ is simply the 1%-quantile of the 250 most recent portfolio returns. In other words, the quantile estimate is based on the second- and third-lowest portfolio returns of the rolling window of 250 trading days.

2.4. Copula-based models

- **Copula-based models** allow for **modelling separately** the univariate distribution functions of the marginal distributions and their copula, i.e. their „dependence structure".
- **Marginal distributions:**
  In our analysis these are the 21 return distributions of the financial assets, modelled as **empirical distributions** based on the 250 most recent volatility-adjusted return observations.
- **Copulas:**
  - **Gaussian copula:** this is the copula implied by a multivariate Gaussian distribution (Gaussian marginal distributions combined with a Gaussian copula).
    It has one (matrix) parameter: the correlation matrix $\mathbf{P}_G$ (capital "Rho")
  - **Student *t* copula:** this is the copula implied by a multivariate Student *t* distribution. It has two parameters: the correlation matrix $\mathbf{P}_t$ and the degrees of freedom ν ("nu", a scalar parameter). The lower ν the stronger is the **tail dependence.** In fact, the Gaussian copula can be considered a special case of the Student *t* copula where $\nu \to \infty$.

2.4. Copula-based models

- **Copula Calibration**
- We use two methods to calibrate copula parameters:
  - The **pseudo-log-likelihood method**
    Genest and Rivest (1993) and McNeil (2005)
    The most widely used calibration method
    Computationally intensive
  - The **method-of-moments (MoM)**

2.4. Copula-based models

- **Pseudo-log-likelihood method**
- A variant of the **maximum likelihood estimation (MLE).** In practice one often maximises the log-likelihood as this is computationally less intensive and the optimisation yields the same results.
- In contrast to the standard MLE, one does not make any assumption on the specific functional form of the marginal distribution functions in the pseudo-log-likelihood method.
- Rather, the observations are transformed into so-called **pseudo-observations** in a first step. The pseudo-observations $u_{i,t}$ for asset $i \in (1, \dots, 21)$ and trading day $t \in (1, \dots, 250)$ are calculated from the volatility-adjusted returns $\tilde{r}_{i,t}$ as

  $$u_{i,t} = \frac{1}{T+1} \cdot \sum_{s=1}^{T} \mathbf{1}_{\tilde{r}_{i,s} < \tilde{r}_{i,t}}$$

  where $\mathbf{1}_{\tilde{r}_{i,s} < \tilde{r}_{i,t}}$ is an indicator function that take on a value of 1 if $\tilde{r}_{i,s} < \tilde{r}_{i,t}$ and a value of 0 otherwise. In our analysis $T = 250$.
- In other words, the rank of the observation is divided by 251 to obtain $u_{i,t}$.

2.4. Copula-based models

- **Pseudo-log-likelihood method**
- For the **Gaussian copula**, the correlation matrix $\mathbf{P}_G$ is calibrated via numerical optimisation as

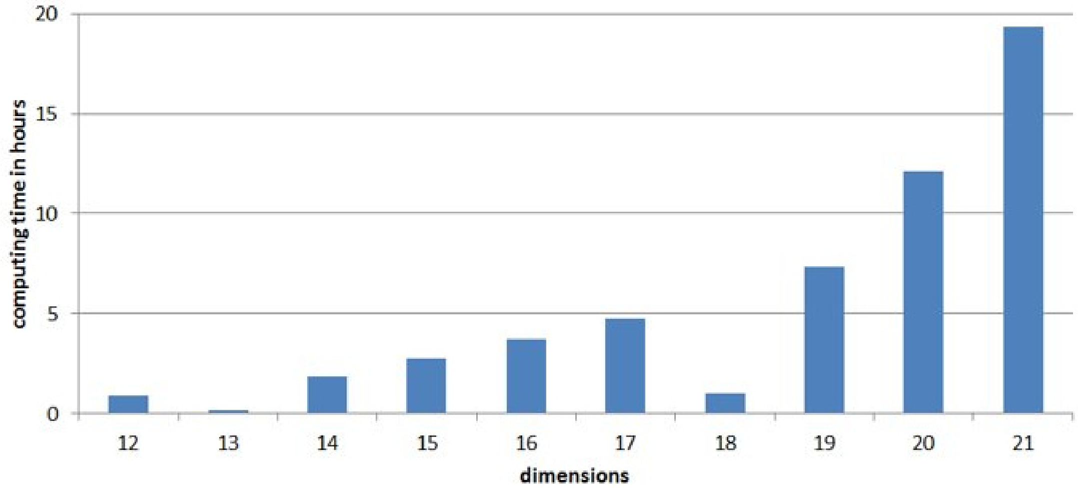
$$\mathbf{P}_G = \operatorname{argmax}_{\mathbf{P}_G} \sum_{t=1}^{250} \left(\ln \phi_{\mathbf{P}_G} \left(\Phi^{-1}(\mathbf{u}_t) + \sum_{i=1}^{21} \ln \left(\frac{1}{\phi\left(\Phi^{-1}(u_i, t)\right)} \right) \right) \right)$$

  with $\ln$ the natural logarithm function, $\phi_{\mathbf{P}_G}$ the probability density function of a multivariate standard normal distribution with parameter (i.e. correlation matrix) $\mathbf{P}_G$, ϕ the density function of a univariate standard normal distribution, Φ^{-1} the quantile function of a univariate standard normal distribution and $\mathbf{u}_t = \left(u_{1,t}, u_{2,t}, \dots, u_{21,t}\right)$ a vector representing the joint pseudo-observations for trading day t.
- We use R-package `copula` and, here, function
  `fitCopula(normalCopula(dim=21, dispstr="un"), method="mpl")`
  to perform the maximum-log-likelihood estimation.
  We refer to the so-calibrated model as model Gauss_MLE.

2.4. Copula-based models

- **Pseudo-log-likelihood method**
- **Student *t* copula**
  Unfortunately, the MLE-calibration of a high-dimensional Student *t* copula is computationally very intensive, i.e. time-consuming.
- Hence, we choose an alternative approach:
  - use $\mathbf{P}_G$ as a substitute for $\mathbf{P}_t$
  - Set $\nu = 3$ without calibration
- We refer to so-calibrated model as model t_MLE.

Small simulation study: Calibration time for Student *t* copula
We simulate random data of 250 sets for different dimensions
(Gaussian copula rvs, only one scenario)



dimensions	12	16	20	21
computing time in h	0.9	3.7	12.1	19.3

data source: Aussenegg and Cech (2012), table 1

2.4. Copula-based models

- **“method-of-moments” (MoM):** Not so computationally intensive.
- **Gaussian copula**
  the elements $\rho_{G,ij}$ of the Gaussian copula parameter $\mathbf{P}_G$ are calibrated as the pairwise Spearman’s rank-correlation coefficient between asset returns i and j, $\rho_{S,ij}$
  $$\rho_{G,ij} = \rho_{S,ij} = \frac{6}{\pi}\arcsin\frac{\rho_{ij}}{2}$$
  where ρ_{ij} is the pairwise (Pearson) correlation coefficient between the pseudo-observations.
- **Student *t* copula**
  the elements $\rho_{t,ij}$ of the Student *t* copula parameter $\mathbf{P}_t$ are calibrated as
  $$\rho_{t,ij} = \sin\left(\frac{1}{2}\pi\rho_{\tau,ij}\right)$$
  where $\rho_{\tau,ij}$ is the pairwise Kendall’s rank-correlation coefficient (Kendall’s tau).
  The MoM allows only the calibration of $\mathbf{P}_t$. We set the second parameter ν equal to 3.

2.4. Copula-based models

- **Copula simulation**
  For our four models (models 3.a, 3.b, 4.a and 4.b) and for every trading day analysed we simulate **one million** 21-dimensional tuples.
  To reduce the variance of the Monte Carlo simulation we additionally construct **antithetic variates** (see e.g. Glasserman, 2003, pp. 205ff.)

  This results in two million simulated values of 21-tuples with

$$\hat{u}_i, d$$

  where $i \in (1, \ldots, 2{,}000{,}000)$ and $d \in (1, \ldots, 21)$

2.4. Copula-based models

- **Copula simulation**
  In a further step, we use the empirical distribution of the 250 most recent volatility-adjusted return observation of every asset to simulate quantiles of asset returns.
- Then, we calculate for every simulated scenario portfolio returns

$$\hat{r}_{pf,s} = \sum_{d=1}^{21} w_d \cdot F_d^{-1}\left(\hat{u}_{i,d}\right)$$

  where w_d are the portfolio-weights and $F_d^{-1}\left(\hat{u}_{i,d}\right)$ are the empirical quantile functions of asset returns.
- Finally, we estimate next day's return quantile as the 1%-quantile of the two million simulated portfolio returns.

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3. Hit tests

- We conduct hit tests for the estimates of the 1%-return quantiles.
- A “hit” (or “exception”) refers to those days where the realised return on trading day $t+1$ is below the quantile-estimate based on data up to trading day t.
- Obviously, in a good model this should happen in roughly 1% of the cases.
- We use two types of hit tests:
  - **Unconditional hit tests**
    consider only the proportion of hits
  - **Conditional hit tests**
    examine the clustering of hits
- For every model we create hit-sequences $\mathbf{I} = (I_1, I_2, \ldots, I_T)$ where $I_t = 0$ if no hit is observed and $I_t = 1$ if a hit is observed on trading day t. Number of hits $n = \sum_{i=1}^{T} I_t$.

3. Hit tests

- **Unconditional hit tests,** $H_0\!: \hat{p} = 0.01$ where $\hat{p} = \frac{n}{T}$
  - hit test based on a binomial distribution (**Campbell 2007**, p.6)
  - **Kupiec test** (Kupiec, 1995) , also known as proportion-of-failure **(PoF)** test

- **Conditional hit tests** additionally test the null hypothesis $H_0\!:\ \tau_{01} = \tau_{11}$ where τ_{01} is the probability of observing a hit on day t ("today") conditional of observing no hit on day $t-1$ ("yesterday"), i.e. $P(I_t = 1|I_{t-1} = 0)$ and $\tau_{11} = P(I_t = 1|I_{t-1} = 1)$.
  - **Christoffersen test** (Christoffersen, 1998)

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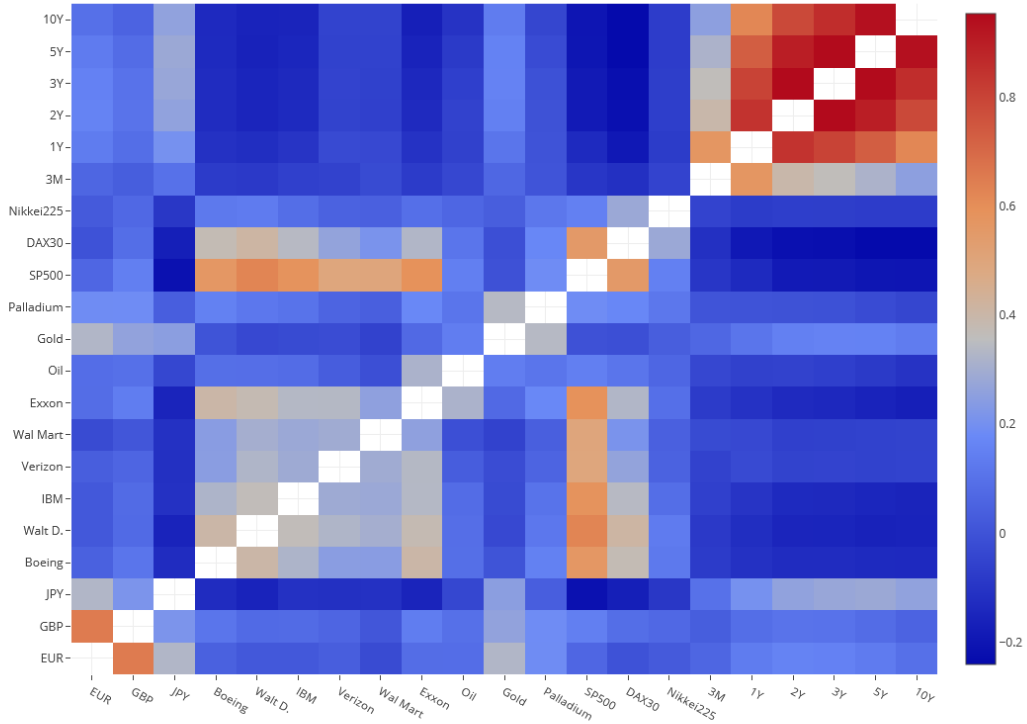
4. Data

- Daily log-returns of **21 financial assets** from 5 January 1990 to 26 November 2024.
- The first 500 tuples are used for model-calibration.
- Quantiles are estimated for the period from 7 January 1992 to 26 November 2024, i.e. for **8,252 trading days**, based on the return observations up to the trading day before.
- The financial assets can broadly be classified into five classes:
    i. Foreign exchange (three assets)
    ii. Blue-chip stocks (six assets)
    iii. Stock-indices (three assets)
    iv. Commodities (three assets)
    v. Fixed-income instruments (six assets)
- We take a USD-investor perspective.

1: EUR	8: Wal Mart	15: Palladium
2: GBP	9: Exxon	16: 3M
3: JPY	10: SP500	17: 1Y
4: Boeing	11: DAX30	18: 2Y
5: Walt D.	12: Nikkei225	19: 3Y
6: IBM	13: Oil	20: 5Y
7: Verizon	14: Gold	21: 10Y

4. Data

- **21 financial assets**
  - Low volatilities for fixed-income, high for stocks, indices, commodities.
  - All returns are leptokurtic and *not* normally distributed (Jarque-Bera).
  - Pearson-correlations range from -0.240 (DAX30, 5Y) to 0.953 (2Y, 3Y).



lowest returns				
12 Mar. 2020	20 Apr. 2020	16 Mar. 2020	9 Mar. 2020	18. Mar 2020
-6,57%	-5.72%	-4.93%	-4.37%	-4.28%

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- **Summary of model results**

Table 3: Number of hits, proportion of hits, unconditional hit tests, and Christoffersen test

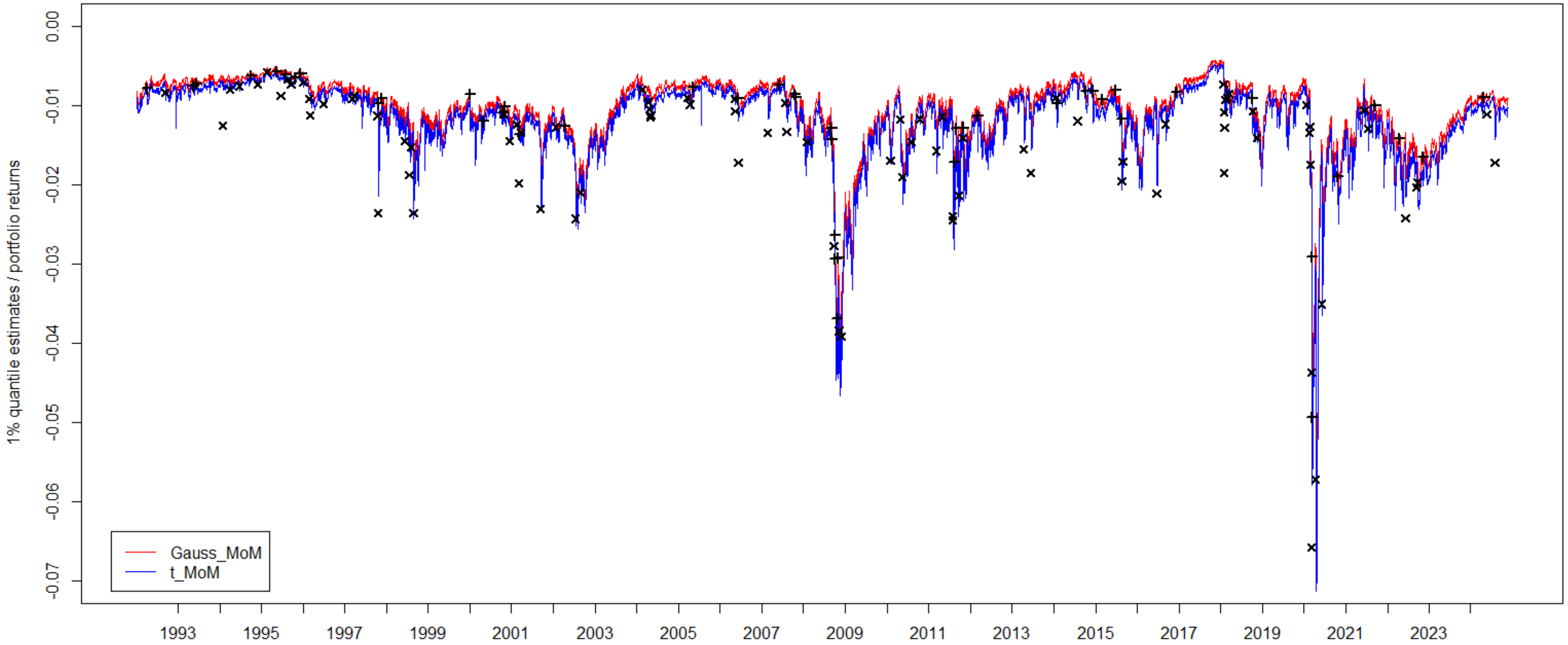
	Variance-Covariance models			Historical Simulation models		Copula based models			
Model	VC_unadj	VC_EWMA	VC_adj	HS_unadj	HS_adj	Gauss_MLE	t_MLE	Gauss_MoM	t_MoM
Panel A: Hits									
no. of hits	169	163	155	134	120	135	91	136	90
$\hat{p}$	2.05%	1.98%	1.88%	1.62%	1.45%	1.64%	1.10%	1.65%	1.09%
$\hat{q}/\hat{q}(VC\_unadj)$	1	0.968	0.971	1.122	1.042	1.004	1.108	1.002	1.110
$\widehat{ES}/\widehat{ES}(VC\_unadj)$	1	0.968	0.971	1.142	1.067	1.008	1.133	1.006	1.135

$\hat{q}/\hat{q}(VC\_unadj)$: the average ratio of the models' quantile estimates $\hat{q}$ in relation to the quantile estimate of *model VC_unadj*

$\widehat{ES}/\widehat{ES}(VC\_unadj)$: the average ratio of the models' estimates for the mean lowest 2.5% return observations $\widehat{ES}$ in relation to the corresponding estimates of *model VC_unadj*

5. Results

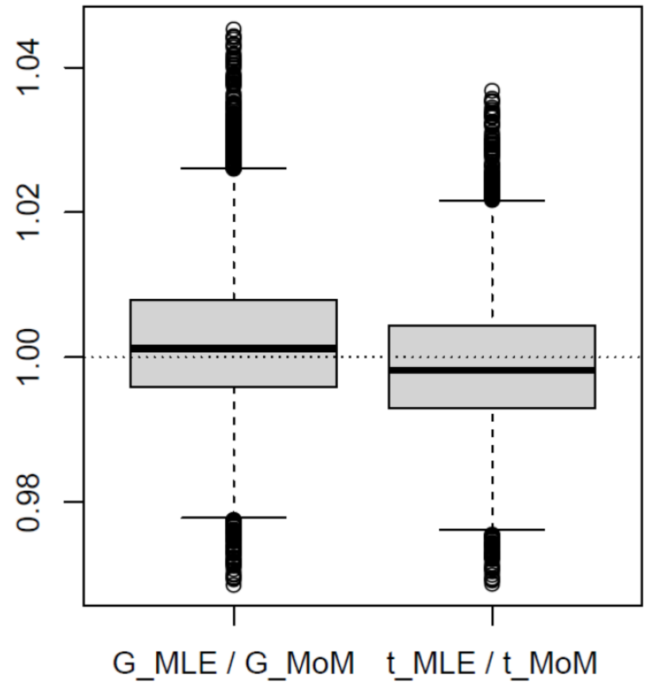
- **Copula-based simulation models, MoM** (models Gauss_MoM, t_MoM)



	Gauss_MoM	t_MoM
n	136	90
$\hat{p}$	1. 65%	1.09%

5. Results

- **Copula-based simulation models**
  (models Gauss_MLE, t_MLE, Gauss_MoM, t_MoM)
  - In all cases the quantile estimate of the meta-Student *t* models are below those of the meta-Gaussian models.
    On average 10.4% (model t_MLE) respectively 10.7% (model t_MoM).
  - Accordingly, the proportion of hits is lower.
  - The results of the MLE-models are similar to those of the MoM-models.



5. Results

- **Unconditional hit tests**

Model	Variance-Covariance models			Historical Simulation models		Copula based models			
	VC_unadj	VC_EWMA	VC_adj	HS_unadj	HS_adj	Gauss_MLE	t_MLE	Gauss_MoM	t_MoM
Panel B: Campbell test									
z	9.57	8.90	8.02	5.70	4.15	5.81	0.94	5.92	0.83
sign.	**	**	**	**	**	**		**	
Panel C: Kupiec test									
LR_{uc}	(a)	(a)	(a)	27.29	15.08	28.28	0.85	29.29	0.67
sign.				**	**	**		**	

null hypothesis can be rejected at the 1% (**) or the 5% (*) significance level.

(a) The Kupiec and the Christoffersen test do not provide results due to the large number of hits and the large number of two hits on two consecutive trading days (T_{11}). Based on the proportion of hits and the number of hits on two consecutive trading days we conclude that the null hypotheses for both tests can be rejected.

→ We cannot reject the null hypothesis for the meta-Student *t* models.

→ We reject the null hypothesis for all other models at the 1% significance level.

5. Results

- **Christoffersen test**

Model	Variance-Covariance models VC_unadj	VC_EWMA	VC_adj	Historical Simulation models HS_unadj	HS_adj	Copula based models Gauss_MLE	t_MLE	Gauss_MoM	t_MoM
Panel D: Christoffersen test									
T_{11}	15	9	11	11	5	8	3	8	3
LR_{ind}	(a)	(a)	(a)	19.22	4.20	9.53	2.67	9.35	2.76
sign.				**	*	**		**	
LR_{cc}	(a)	(a)	(a)	46.51	19.28	37.81	3.52	38.63	3.42
sign.				**	**	**		**	

null hypothesis can be rejected at the 1% (**) or the 5% (*) significance level.

[a] The Kupiec and the Christoffersen test do not provide results due to the large number of hits and the large number of two hits on two consecutive trading days (T_{11}). Based on the proportion of hits and the number of hits on two consecutive trading days we conclude that the null hypotheses for both tests can be rejected.

→ We cannot reject the null hypotheses for the meta-Student *t* models.

5. Results

- **Expected Shortfall:**
  in addition to the 1% return quantile (→ 99% Value-at-Risk VaR), we calculate the mean return below the 2.5% return quantile (→ 97.5% Expected Shortfall ES).
  On average the two values do not deviate strongly from each other.

Table 4: Ratio of 97.5% Expected Shortfall (ES) and 99% VaR

	Variance-Covariance models	Historical Simulation models		Copula based models			
Model		HS_unadj	HS_adj	Gauss_MLE	t_MLE	Gauss_MoM	t_MoM
Mean	1.0050	1.0263	1.0305	1.0096	1.0097	1.0279	1.0280
Median	1.0050	1.0145	1.0216	1.0092	1.0092	1.0288	1.0288
Min	1.0046	0.8655	0.8701	0.9909	0.9920	1.0030	1.0038
Max	1.0055	1.3767	1.3077	1.0844	1.0873	1.0880	1.0891

Notes: This table presents the ratio of 97.5% ES/99% VaR. Mean and median numbers above (below) one indicate that 97.5% ES estimates are larger (smaller) than the 99% VaR estimates.

5. Results

- **Expected Shortfall:**

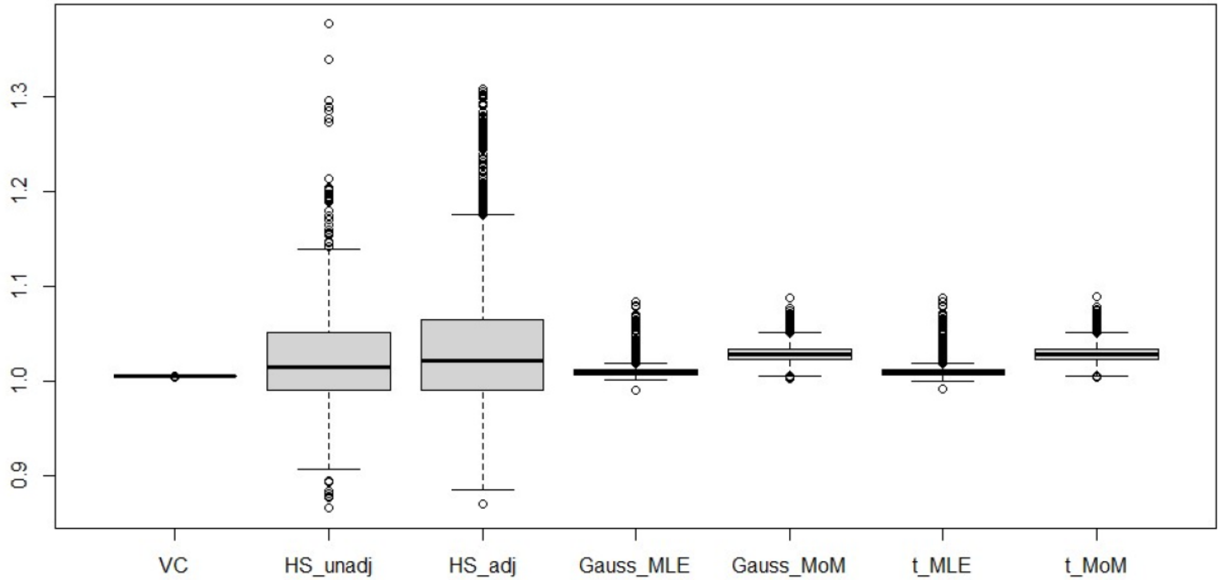


Figure 3: Ratio of 97.5% Expected Shortfall (ES) and 99% VaR

Notes: This figure shows the ratio of 97.5% Expected Shortfall (ES) and 99% VaR. For values above (below) 1 the 97.5% ES is larger (smaller) than the 99% VaR. The median ratio (thick line in the middle) reveals that using the 97.5% ES is on average quite comparable with the 99% VaR and on average only about 0.5 to 3% above the 99% VaR.

5. Results

- **Computing time:**
  - The MLE- and MoM-models yield similar results.
  - However, the latter are far less computationally intensive.

Table 5: Computing time for copula-based models

Panel A: Real data - Daily computing time in seconds, average over all 8,252 trading days						
		MLE Estimation	MoM Estimation		Simulation	
	Estimation of EGARCH volatility adj. returns	Gaussian and Student *t* copula	Gaussian copula	Student *t* copula	Gaussian copula	Student *t* copula
Time (seconds)	7.6	152	0.03	0.32	11.66	20.16

5. Results

- **Computing time:**
  - The computational advantage of MoM models also holds for higher dimensions
  - Simulation study:
    - Simulate 250 d-dimensional realisations of a Gaussian copula with pairwise correlation parameter $\rho_{i,j} = 0.5\ \forall i,j$ where $i \neq j$
    - Estimate copula parameters and simulate copulas (10^6 realisations and 10^6 antith. v.)

Panel B: Simulation – Computing time for one copula estimation and copula simulation in seconds for different dimensions

	MLE Copula Estimation	MoM Copula Estimation		Copula Simulation	
Dimensions	Gaussian and Student *t*	Gaussian	Student *t*	Gaussian	Student *t*
10	7.28	0.01	0.12	2.56	5.15
20	60.62	0.02	0.45	5.32	10.54
50	2,561.67	0.80	3.82	14.82	27.71
100	72,722.11	12.67	24.58	34.54	59.92
200		269.77	326.26	94.72	147.37
300		1,451.70	1,566.16	166.89	247.87
400		4,649.56	4,922.28	267.29	374.23

Notes: The calculations are performed on a Windows 10 desktop computer with an Intel Core i7-5930K 3.5 GHz CPU and a 32GB memory. In Panel B the MLE estimation time for dimensions above 100 is not available due to the exponential growth in computing time.

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6. Conclusion

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- We analyse the accuracy of nine models to estimate the one-day 1% return quantile.
- Five benchmark models
  (three different variance-covariance models and two historical simulation models)
- Four copula models that combine volatility-adjusted returns with
  a Gaussian or a Student *t* copula.
- Copula calibration: pseudo-log-likelihood (MLE) or method-of-moments (MoM)
- These models generate the daily return distribution of a 21-dimensional equally weighted mixed asset portfolio.
- **The meta-Student *t* model calibrated with the method of moments outperforms** the other models.
  - It shows the best accuracy regarding the hit proportion and
    the independence of hit observations and
  - it is computationally efficient

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Thank you for your attention!

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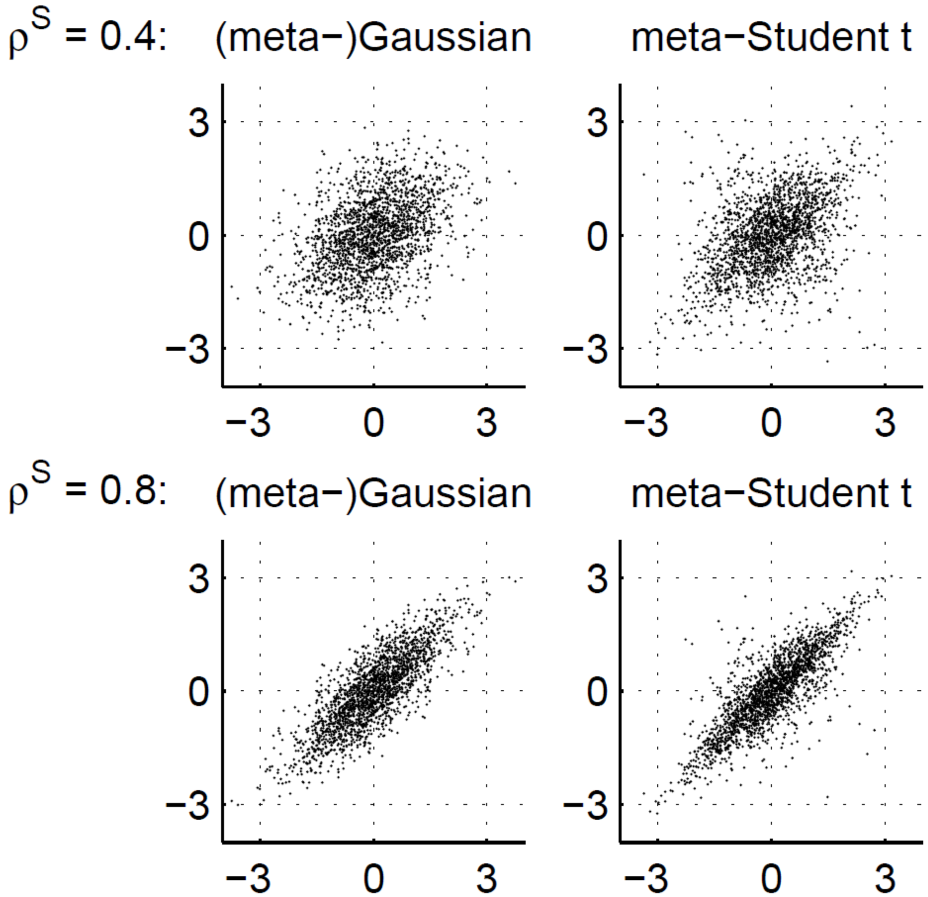
Appendix: Copula-based models

- We use two types of copulas:
  - **Gaussian copula**
  - **Student *t* copula**
- The Student t copula displays positive *"tail dependence"*: It assigns a higher probability to joint extreme events than does the Gaussian copula.
- The plots show standard normal distributions combined by Gaussian and Student *t* copulas.



Source: Cech (2006)