Estimating Time-Varying Risk Aversion from Option Prices and Realized Returns

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Quantitative Finance, forthcoming

Risk aversion

- Risk aversion: central role in asset pricing, but not directly observable
- Interesting for practitioners: e.g., market timing, TAA.
- Interesting for academics: e.g., test hypotheses about drivers of RA.
- Literature:
  - Estimate risk aversion of individuals
  - Estimate risk aversion of the market
- **This paper: Estimate risk aversion of the market from option prices and realized returns**

RND, PD, and SDF

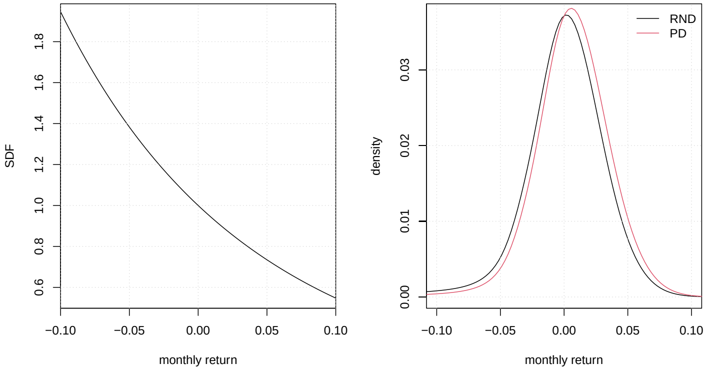

Figure: RND, PD, and SDF estimated on June 30, 2021 from a 3-year time window. Left: SDF estimated using the power utility (PU) specification. Right: RND estimated from option prices (black) and the resulting PD forecast (red).

Related Literature

- Information content of RNDs from option prices (Jackwerth, 2004; Figlewski, 2010)
- Infer the SDF from RNDs and PDs estimated via historical returns and kernel densities (Ait-Sahalia and Lo, 2000; Jackwerth, 2000; Rosenberg and Engle, 2002; Barone-Adesi et al., 2008; Grith et al., 2013)
    - Pricing kernel puzzle: SDF increases on parts of its domain. Overview: Cuesdeanu and Jackwerth (2018)
- Alternative approach (Bliss and Panigirtzoglou, 2004; Kostakis et al., 2011): Assume a utility function to transform RND into a PD based on best fit of physical densities to subsequently realized returns
    - If these utility functions imply a monotonically decreasing SDF, this "assumes away" the pricing kernel puzzle by construction!

Methodology

- We largely follow Bliss and Panigirtzoglou (2004), except for the construction of RNDs (spline with 4 df, following Figlewski (2018)).
- One-month options, non-overlapping time periods
- Power utility and two more flexible SDFs (incl. pricing kernel puzzle)
- Evaluations in- and out-of-sample

SDF specifications

Three different (non-normalized) SDFs:

- Power utility (PU):

$$\xi_t'(R) = e^{-\gamma_t R}. \tag{1}$$

- Sum of discount functions (LIN):

$$\xi_t'(R) = \exp(-a_t^2 R) + \exp(-b_t^2 R) + \exp(-c_t^2 R) + d_t^2. \tag{2}$$

- Cubic polynomial (POLY):

$$\xi_t'(R) = -a_t^2 R^3 + b_t^2 R^2 + c_t R + d_t. \tag{3}$$

SDF specifications: shapes

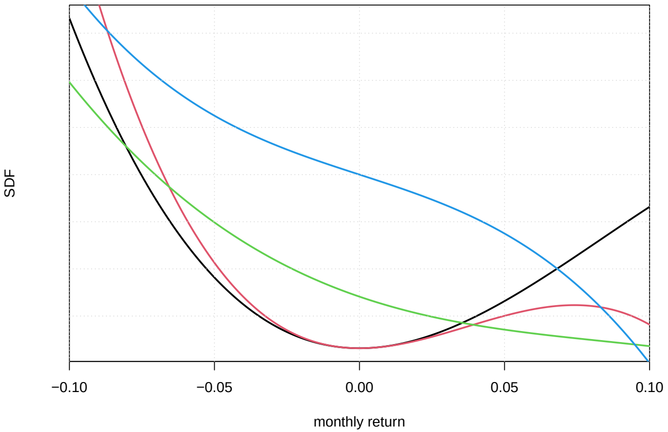

Figure: Examples for different shapes which can be modeled using the polynomial SDF specification (POLY) from equation (3). In addition to monotonically decreasing, left-curved SDFs (green), it also allows for decreasing but convex-concave shapes (blue), wave-like (red) and U-shaped (black) SDFs.

SDF estimation for a fixed time window

- How to estimate the parameters of equations (1) to (3)?
- Bliss and Panigirtzoglou (2004) evaluate the likelihood of the PDs resulting from the estimated SDFs against subsequent returns in-sample
- Comparability of the quality of the estimated PDs across time: use an inverse transformation of R_{i+1}, the realized return at time $i+1$, using the PD estimated at time i (Rosenblatt, 1952).
- Under the null hypothesis that the estimated PDs are equal to the true PDs, $\hat{f}_i(\cdot) = f_i(\cdot)\ \forall i$, the inverse probability transformations of the realizations,

$$y_i = \int_{-\infty}^{R_{i+1}} \hat{f}_i(u)du, \tag{4}$$

  will be independently and uniformly distributed.
- Jointly test independence and uniformity (Berkowitz, 2001)

Time windows and weighting schemes used in the SDF estimation

1. In-sample (using all available data)
2. Rolling windows as in Kostakis et al. (2011)
3. Exponential weighting with weights depending on current level of implied volatility (to avoid well-known problems of rolling windows)

Higher flexibility may lead to noisier estimates → out-of-sample evaluations

Estimation of risk aversion from the SDFs

- Power utility (PU): estimate RRA γ_t directly
- LIN and POLY: Calculate a proxy for RA as the value for γ_t in the corresponding power utility SDF from equation (1) which equalizes the expected returns in the PDs from PU and those from LIN/POLY

Data

- Monthly data from equity options on the S&P 500
- IV surfaces in the delta dimension
- Sample period: Dec. 31, 2007 – Aug. 31, 2021 ($t = T$), with 165 observations in total.

Entire dataset and rolling windows

	SDF	$p(LR_3)$	RMSE	MAFE	exc. ret.	std. dev.	skewness
	PU	87.39	4.78	3.35	14.89	14.48	–45.14
ALL	LIN	96.87	4.65	3.32	14.35	14.37	–58.43
	POLY	98.04	4.64	3.32	14.23	14.29	–61.43
	PU	38.46	3.80	2.80	12.50	12.22	–42.32
3Y	LIN	15.76	3.78	2.77	13.19	11.74	–40.06
	POLY	31.86	3.76	2.83	14.58	12.06	–67.51
	PU	19.84	3.68	2.70	13.07	11.57	–42.77
5Y	LIN	10.64	3.68	2.67	13.22	11.16	–41.92
	POLY	14.03	3.69	2.71	15.26	11.65	–48.61

Table: Estimation of physical densities from options data. Three panels with results from three SDF specifications each, where PU is power utility from equation (1), LIN is the mixture of discount functions from equation (2), and POLY is the polynomial SDF from equation (3). Reported values are averages across the in-sample evaluations for the estimation using the entire dataset and across the out-of-sample evaluations for the 3Y and 5Y estimations. All values in percent.

Expanding windows with exponentially weighted estimations

	$\lambda(\overline{\sigma}^{\text{ATM}})$	RMSE	MAFE	exc. ret.	std. dev.	skewness
PU	98.5	3.72	2.73	9.52	11.81	–55.26
LIN	98.5	3.71	2.70	10.27	11.58	–57.35
	97.5	3.72	2.72	12.44	11.85	–57.94
	98.0	3.69	2.70	12.10	11.76	–60.39
POLY	98.5	**3.63**	**2.67**	11.82	11.79	–62.88
	99.0	3.65	2.68	10.68	11.68	–64.71
	99.5	3.67	2.70	9.43	11.62	–68.16

Table: Estimation of physical densities from options data. Three panels with results from one SDF specification each, estimated from expanding windows with exponential weighting of past observations using different decay factors. All values in percent.

Time-varying and pro-cyclical risk aversion

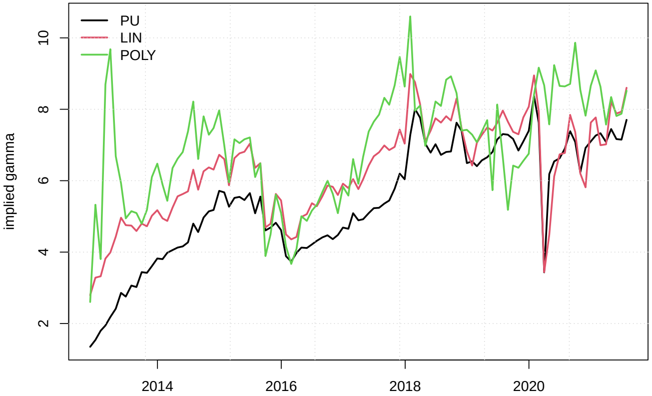

Figure: Time-varying risk aversion γ_t estimated from expanding windows with exponential weighting. Risk aversion implied from different SDFs for a decay factor level of $\lambda(\overline{\sigma}^{\text{ATM}}) = 98.5\%$: PU (black), LIN (red), and POLY (green).

Is risk aversion really pro-cyclical?

A constant or even increasing risk aversion in times of crises (counter-cyclical risk aversion) would lead to very high levels of expected returns:

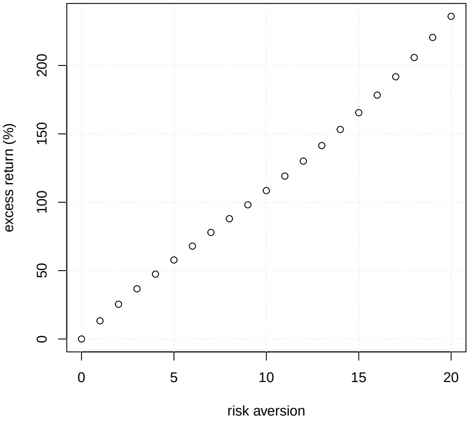

Figure: Expected excess return from one-month forecasts made on March 31, 2020, using PU with different values for the risk aversion γ.

Time variation in expected excess returns

Expected returns increase when the risk aversion increases and/or when the implied volatility increases:

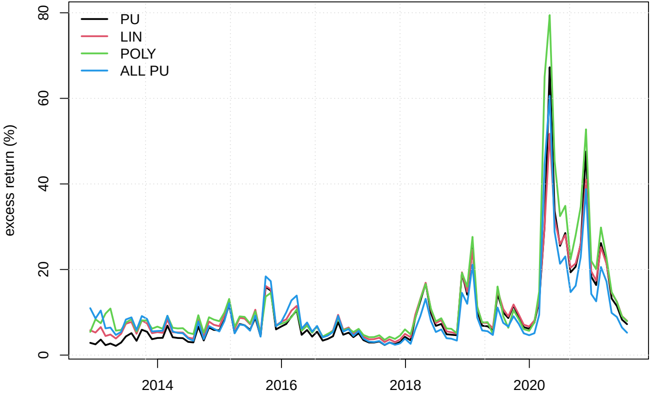

Figure: Expected excess returns implied by different SDFs estimated from expanding windows using exponential weighting with a decay factor level of $\lambda(\overline{\sigma}^{\text{ATM}}) = 98.5\%$, and estimated from all data (ALL PU).

Variance risk premia

Christoffersen et al. (2021) show that under certain assumptions the price of coskewness risk corresponds to the market variance risk premium, i.e. the difference between physical and risk-neutral variance:

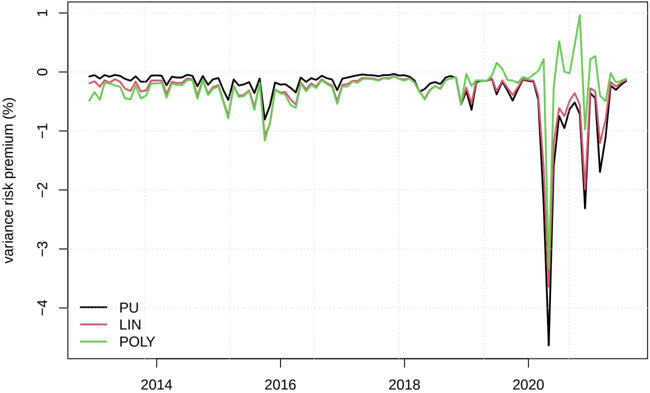

Figure: Market variance premium, in percent, for different SDF specifications estimated from expanding windows using exponential weighting with a decay factor level of 98.5%.

Realized vs. forecast volatility

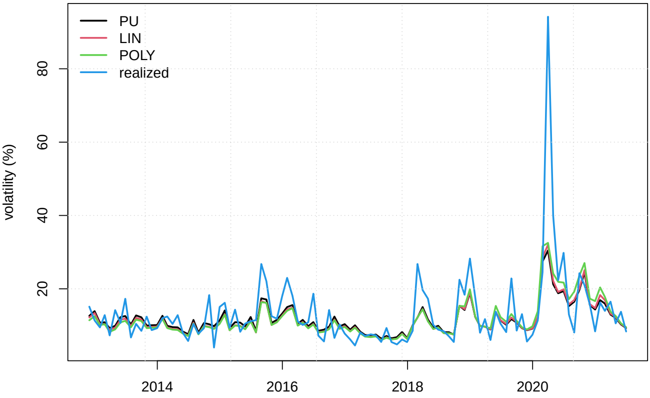

Figure: Realized volatility (blue) vs. out-of-sample volatility forecasts from different SDF specifications (PU: black, LIN: red, POLY: green) estimated from expanding windows using exponential weighting with a decay factor level of 98.5%.

Pricing kernel puzzle

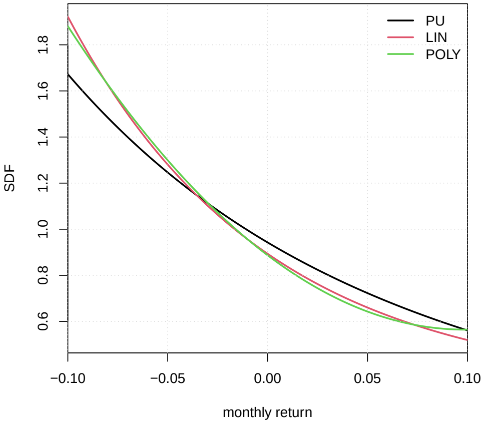

Figure: Optimal SDFs estimated on the entire dataset (ALL), corresponding to the top panel in Table 1: PU (black), LIN (red) and POLY (green).

Pricing kernel puzzle

	SDF shape		
	0	1	2
3Y	35.7	60.5	3.9
5Y	42.9	52.4	4.8
$\lambda = 98.5$	32.4	67.6	0.0

Table: Distribution of different SDF shapes when estimating the POLY SDF on three- (3Y) and five-year (5Y) rolling windows, and on an expanding window with exponential weighting (decay factor $\lambda = 98.5$). Values denote the fraction (in percent) of the respective shape relative to the total number of estimated SDFs. 0 denotes a monotonically decreasing shape, 1 is U-shaped, and 2 is wave-like. The numbers used to denote the different SDF shapes coincide with the number of sign changes in the slope of the estimated SDFs.

Pricing kernel puzzle

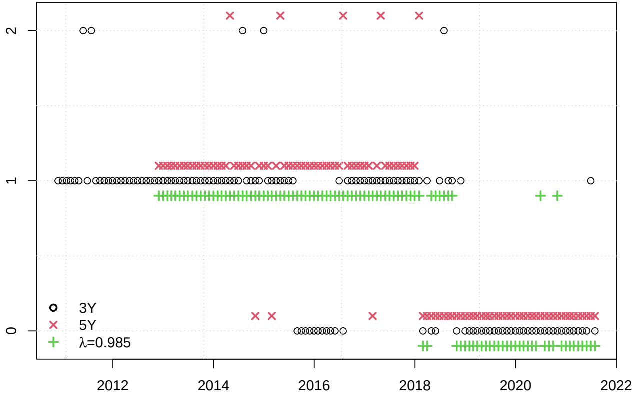

Figure: POLY SDF shapes across time, estimated from 3-year (black) and 5-year (red) rolling windows and from expanding windows with exponential weighting (green): 0 indicates a monotonically decreasing SDF, 1 a U-shaped and 2 a wave-like SDF. Expanding window estimates and 5Y-rolling window estimates start after a run-in period of five years, 3Y-rolling window estimates start two years earlier.

Conclusion

- Different models for estimating time-varying risk aversion from option prices: PU, LIN, POLY
- In-sample: Higher flexibility is better. Optimal SDF (POLY) is steeper and more curved than PU
- Rolling windows: Higher stability of PU sometimes outweighs its lower flexibility
- Exp. weighting: POLY is optimal
- Until early 2018, optimal SDFs are mostly U-shaped, since then, they decrease monotonically → the pricing kernel puzzle is present in the first part of our sample, but has vanished since 2018
- Levels of risk aversion are in line with the literature
- Pronounced time variation in risk aversion, exp. excess returns, and market variance risk premium
- We find pro-cyclical risk aversion (in line with closely related literature, but in contrast to many other approaches)

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