# Option-implied Skewness and the Cross Section of Foreign Exchange Returns

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37th AWG Workshop, September 23, 2022

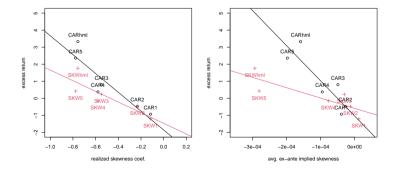
### Intro

- 1. Carry is most prevalent factor in FX-literature and it is based on the fact that currencies tend to depreciate less than what is implied by their interest rate differential (Fama, 1984).
- 2. It is still debated what risks carry returns compensate. One popular explanation is crash risk:
  - Brunnermeier et al. (2008), Farhi et al. (2009) or Burnside et al. (2011) are often cited in this context.
  - All of them attribute a part of the carry risk premium to crash risk and argue/measure with the use of option prices (prices of risk reversals or skewness).
  - Jurek (2014) however, shows that crash-hedged portfolios still have statistically significant returns and estimates only 1/3 of the carry risk premium to be crash risk compensation.



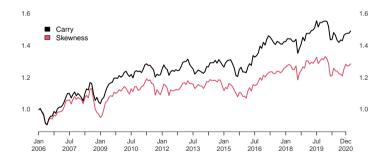
Risk and return of carry and model free option-implied skewness (Schneider and Trojani, 2015) ranked quintile portfolios:

- CAR and SKW portfolios seem to exhibit differrent returns for similar realized skewness coefficient.
- Ex ante, carry portfolios seem to be less risky, yet have higher returns
- Existing research looks at the CAR parts of the plots for inference.



## Idea II

Portfolios constructed from opition-implied skewness (SKW) compared with carry (CAR): SKW and CAR are highly correlated with CAR performing better.



2006-01-31 / 2021-03-31

# Contribution

Investigate the interplay of carry and option-implied skewness in the context of portfolio construction. Different to past literature this paper

- 1. uses option data only as signals as opposed to investments/hedges. (increased sample!)
- 2. Combining these signals with traditional carry, I construct two novel strategies:
  - a 'skew-neutral' carry (RDF) where high-risk, low-yielding currencies are shorted and high-yielding, low-risk currencies are bought. When added to a pricing model, this factor is priced during the sample period whereas carry is not.
  - a 'crash-hedged' carry (CAR<sub>hedged</sub>) where high-risk, low-yielding currencies are shorted against the long leg of the carry factor. This strategy makes a strong empirical argument against crash-based explanations for the carry risk premium.

# Idea III

High correlation of returns is to be expected, since the portfolios are very similiar (rank-correlation • appendix) of CAR and SKW ranking is high), but why are there differences?

- There seem to be currencies with high interest and comparatively low negative skewness and vice versa.
- Portfolios consisting of such currencies might have interesting properties. But how to construct them?
  - Double sorts are suboptimal because of high correlation/low number of assets some undesirable currencies are selected and these enter portfolios with equal weight. • appendix
  - Solution: weighting scheme based on rank-difference (RDF):

$$w_{c,t}^{RDF} = \kappa_t \left[ \operatorname{rank}(CAR_{c,t}) - \operatorname{rank}(-SKEW_{c,t}^{\mathbb{Q}}) \right]$$
(1)

# Rank difference based weighting • table with example

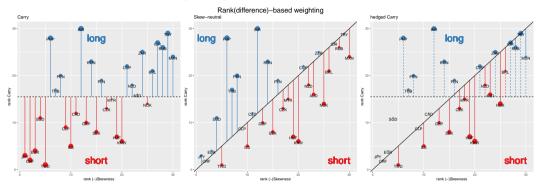


Figure: Illustration of weighting of different portfolios (March 2020). Left graph shows the Carry (*CAR*) portfolio with rank-based weights as in Asness et al. (2013). The middle graph shows the skew-neutral porfolio based on rank-differences (*RDF*) and the right graph illustrates the weights of the hedged carry, which is compromised of the long leg of *CAR* and short leg of *RDF*. Final weights are proportional to vertical lines. Blue dots represent positive weights an red dots negative weights. • charts & stats

Advantages of rank-difference based weighting compared to double-sort

### No empty portfolios

- Currencies close to 45-degree line (low discrepancy) have very small impact.
- Currencies that have a particularly high discrepacy in their signals are overweighted.
- Currencies that have a low skewness but even lower carry are also added to shorts.
- ▶ High carry currencies that have even higher skewness are (partly) neutralized!

| Panel | A: | returns | all | currencies |
|-------|----|---------|-----|------------|
|-------|----|---------|-----|------------|

|         | RD    | F C    | ARhedged   | CAR     | SKW      | VAL     | МОМ    | VRP    | USD    |
|---------|-------|--------|------------|---------|----------|---------|--------|--------|--------|
| mean    | 1.6   |        | 2.92       | 2.66    | 1.68     | 1.83    | 0.34   | 0.19   | 1.15   |
| sd      | 5.0   | 7      | 5.42       | 6.57    | 7.26     | 5.40    | 5.95   | 4.62   | 7.93   |
| skew    | 0.0   | 3      | 0.10       | -0.15   | -0.19    | 0.00    | 0.13   | 0.06   | -0.13  |
| kurt    | -0.2  | 0      | -0.14      | 0.08    | 0.13     | 0.03    | 0.18   | 0.08   | -0.15  |
| maxDD   | -7.9  | 7      | -10.11     | -11.30  | -16.08   | -12.24  | -23.43 | -18.51 | -24.33 |
| SR      | 0.3   | 2      | 0.54       | 0.40    | 0.23     | 0.34    | 0.06   | 0.04   | 0.15   |
| Panel B | : spo | t ret  | urns all c | urrenc  | ies      |         |        |        |        |
|         | RDF   | C/     | Rhedged    | CAR     | SKW      | VAL     | MOM    | VRP    | USD    |
| mean    | -1.61 | L      | -3.45      | -5.03   | -4.12    | 1.78    | -0.38  | 1.99   | -1.39  |
| sd      | 5.10  | )      | 5.40       | 6.56    | 7.25     | 5.41    | 5.96   | 4.67   | 7.93   |
| skew    | -0.03 | 3      | 0.03       | -0.18   | -0.21    | -0.00   | 0.21   | 0.08   | -0.18  |
| kurt    | -0.22 | 2      | -0.14      | 0.09    | 0.14     | 0.03    | 0.23   | 0.08   | -0.13  |
| SR      | -0.32 | 2      | -0.64      | -0.77   | -0.57    | 0.33    | -0.06  | 0.43   | -0.18  |
| Panel C | corre | elatio | ns all cur | rencies |          |         |        |        |        |
|         |       | RDF    | CARhedg    | ed CA   | R SKW    | ' VAL   | MOM    | VRP    | USD    |
| RI      | DF    | 1.00   | 0.6        |         | LO -0.35 | 0.21    | -0.36  | -0.08  | -0.47  |
| CARhed  | ged   | 0.62   | 1.0        | 0 0.7   | 79 0.46  | 6 -0.19 | 0.03   | -0.13  | 0.08   |
| C       | 4R    | 0.10   | 0.7        | 79 1.0  | 0.88     | -0.38   | 0.16   | -0.15  | 0.50   |
| SK      | - W   | 0.35   | 0.4        | 46 0.8  | 38 1.00  | -0.45   | 0.32   | -0.12  | 0.69   |
| ν       | ΆL    | 0.21   | -0.1       | L9 -0.3 | 38 -0.45 | 5 1.00  | -0.25  | 0.14   | -0.32  |
| МС      |       | 0.36   | 0.0        | 0.1     | 16 0.32  | -0.25   | 1.00   | 0.09   | 0.23   |
|         |       | 0.08   | -0.1       | L3 -0.1 | L5 -0.12 | 2 0.14  | 0.09   | 1.00   | 0.24   |
| U.      | SD -  | 0.47   | 0.0        | 08 0.5  | 50 0.69  | -0.32   | 0.23   | 0.24   | 1.00   |

The table illustrates summary statistics for the skew-neutral strategy based on rank differentials (RDF), crash-hedged carry CAR<sub>hedged</sub> (and various FX-factor strategies. CAR, SKW, VAL (Asness et al., 2013) MOM (Menkhoff et al., 2012) and VRP (Della Corte et al., 2016) are constructed with rank-based weights like in Asness et al. (2013) and USD is an equal weighted portfolio of all currencies against the US-Dollar. The statistics are annualized and include monthly returns from April 2006 until March 2021.

# Summary statistics II

CAR<sub>hedged</sub>:

- ▶ Is highly correlated to carry (0.79).
- Adds 0.14 to the Sharpe ratio of carry (lower volatility with slightly higher return).
- ▶ Has positive skewness coefficient, lower exc. kurtosis and maximum drawdown.
- Spot returns suggest that the initial forward discount of the portfolio is smaller but also the average loss in spot returns is smaller (and crucially, crashes in the spot seem to be avoided).
- $\Rightarrow$  carry is not compensation for crash risk? Skew-neutral portfolio (*RDF*) is uncorrelated to carry, has similar dynamics as *CAR*<sub>hedged</sub> but worse summary stats.

### What about risk-adjusted returns?

# Spanning regressions RDF

|                     | RDF     | RDF     | RDF     | RDF            | RDF     | RDF            | RDF            | RDF       |
|---------------------|---------|---------|---------|----------------|---------|----------------|----------------|-----------|
| (Intercept)         | 1.643   | 1.434   | 1.584   | 2.257*         | 1.661   | 1.841*         | 1.002          | 1.188     |
|                     | (1.021) | (1.088) | (1.121) | (1.089)        | (1.068) | (0.912)        | (0.803)        | (0.838)   |
| CAR                 |         | 0.079   |         |                |         |                | 0.353***       | 0.437***  |
|                     |         | (0.066) |         |                |         |                | (0.065)        | (0.057)   |
| МОМ                 |         |         | 0.176*  |                |         |                |                | 0.087*    |
|                     |         |         | (0.072) |                |         |                |                | (0.041)   |
| VAL                 |         |         |         | $-0.335^{***}$ |         |                |                | -0.259*** |
|                     |         |         |         | (0.083)        |         |                |                | (0.069)   |
| VRP                 |         |         |         |                | -0.093  |                |                | 0.194*    |
|                     |         |         |         |                | (0.084) |                |                | (0.077)   |
| USD                 |         |         |         |                | . ,     | $-0.297^{***}$ | $-0.445^{***}$ | -0.445*** |
|                     |         |         |         |                |         | (0.038)        | (0.046)        | (0.048)   |
| R <sup>2</sup>      | 0.000   | 0.010   | 0.043   | 0.127          | 0.007   | 0.218          | 0.373          | 0.489     |
| Adj. R <sup>2</sup> | 0.000   | 0.005   | 0.037   | 0.123          | 0.002   | 0.213          | 0.366          | 0.475     |
| Num, obs.           | 182     | 182     | 182     | 182            | 182     | 182            | 182            | 182       |

 $^{***}p < 0.001; \ ^{**}p < 0.01; \ ^{*}p < 0.05$ 

- RDF offers insignificant alpha compared to all factors jointly.
- More than half of the variation in *RDF*-returns remains unexplained in the full multivariate regression.
- CAR on its own is an insignificant explanatory variable but as USD is added it becomes highly significant.

# Spanning regressions RDF<sub>short</sub>

|                     | $RDF_{short}$ | RDF <sub>short</sub> | RDF <sub>short</sub> | $RDF_{short}$ | $RDF_{short}$ | RDF <sub>short</sub> | RDF <sub>short</sub> | RDF <sub>short</sub> |
|---------------------|---------------|----------------------|----------------------|---------------|---------------|----------------------|----------------------|----------------------|
| (Intercept)         | -1.178        | -2.507               | -1.021               | -1.979        | -1.263        | $-1.945^{**}$        | $-1.271^{*}$         | $-1.255^{*}$         |
|                     | (2.522)       | (2.369)              | (2.435)              | (2.478)       | (2.465)       | (0.763)              | (0.666)              | (0.639)              |
| CAR                 |               | 0.499***             |                      |               |               |                      | -0.283***            | $-0.331^{***}$       |
|                     |               | (0.135)              |                      |               |               |                      | (0.051)              | (0.044)              |
| MOM                 |               |                      | $-0.467^{**}$        |               |               |                      |                      | -0.016               |
|                     |               |                      | (0.166)              |               |               |                      |                      | (0.029)              |
| VAL                 |               |                      |                      | 0.436*        |               |                      |                      | 0.069                |
|                     |               |                      |                      | (0.203)       |               |                      |                      | (0.040)              |
| VRP                 |               |                      |                      |               | 0.448*        |                      |                      | $-0.157^{**}$        |
|                     |               |                      |                      |               | (0.191)       |                      |                      | (0.056)              |
| USD                 |               |                      |                      |               |               | 1.151***             | 1.270***             | 1.296***             |
|                     |               |                      |                      |               |               | (0.034)              | (0.031)              | (0.035)              |
| R <sup>2</sup>      | 0.000         | 0.115                | 0.083                | 0.060         | 0.046         | 0.901                | 0.928                | 0.934                |
| Adj. R <sup>2</sup> | 0.000         | 0.110                | 0.078                | 0.054         | 0.041         | 0.900                | 0.927                | 0.933                |
| Num. obs.           | 182           | 182                  | 182                  | 182           | 182           | 182                  | 182                  | 182                  |

 $^{***}p < 0.001; \ ^{**}p < 0.01; \ ^{*}p < 0.05$ 

- RDF<sub>short</sub> offers significant alpha compared to all factors jointly although it does not always produce significant alpha individually.
- USD is the most important explanatory variable as it explains most of the variation. The coefficient is larger (> 1) than for the long part.
- i.e. shorting RDF<sub>short</sub> against the long leg of CAR would (over)hedge its USD exposure while adding Carry exposure and (significant) alpha.

- 1. Kraus and Litzenberger (1976) and Harvey and Siddique (2000) argue that investors care about systematic skewness.
- 2. Lettau et al. (2014) show that carry trades exhibit systematic downside-risk. They are also connected to coskewness (Dobrynskaya, 2014).
- 3. For equities, Schneider et al. (2020) show that exposure to coskewness can explain low-risk anomalies using option-implied skewness as a proxy for coskewness.

CAR<sub>hedged</sub> and RDF could be explained by systematic skewness?

# Crash-risk regressions

|                     | CAR     | RDF     | $CAR_{hedged}$ | CAR      | RDF            | $CAR_{hedged}$ |
|---------------------|---------|---------|----------------|----------|----------------|----------------|
| (Intercept)         | 0.003   | 0.005*  | 0.004          | 0.001    | 0.003**        | 0.002*         |
|                     | (0.004) | (0.002) | (0.004)        | (0.001)  | (0.001)        | (0.001)        |
| Mkt <sub>down</sub> | 0.226** | -0.047  | 0.060          |          |                |                |
|                     | (0.081) | (0.050) | (0.078)        |          |                |                |
| Mkt                 | . ,     | , ,     | . ,            | 0.176*** | $-0.107^{***}$ | 0.021          |
|                     |         |         |                | (0.030)  | (0.027)        | (0.031)        |
| $e_{Mkt}^2$         |         |         |                | -0.251   | -0.220         | -0.056         |
|                     |         |         |                | (0.361)  | (0.215)        | (0.281)        |
| R <sup>2</sup>      | 0.164   | 0.015   | 0.017          | 0.199    | 0.099          | 0.005          |
| Adj. R <sup>2</sup> | 0.150   | -0.001  | 0.000          | 0.190    | 0.089          | -0.007         |
| Num. obs.           | 62      | 62      | 62             | 182      | 182            | 182            |

 $^{***}\rho < 0.001; \ ^{**}\rho < 0.01; \ ^{*}\rho < 0.05$ 

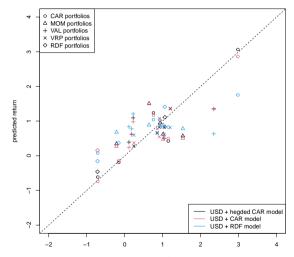
- CAR has (downside) equity market exposure.
- ▶ *RDF* has negative (downside) market exposure.
- CAR<sub>hedged</sub> has neutral (downside) market exposure.

# Asset pricing models with RDF and CAR<sub>hedged</sub>

**Table:** This table includes results from estimating asset pricing models of the form  $E_t[M_{t+1}R_{t+1}^i] = 0$  with a linear stochastic discount factor  $M_{t+1} = 1 - b(f_{t+1} - \mu)$  via GMM. Test assets are quintile portfolios of  $CAR_i$ ,  $MOM_i$ ,  $VAL_i$ ,  $VRP_i$  and the long and short portfolio of RDF. As factors USD, CAR, RDF and  $CAR_{hedged}$  are used. Estimates for market prices of risk  $\lambda$  and factor loadings b, as well as  $R^2$ , square-root of mean squared errors, and p-values of  $\chi^2$  tests are reported. The models include annualized monthly excess returns from January 2006 until March 2021. Standard errors are adjusted according to Newey and West (1987) using an optimal number of lags according to Andrews (1991).

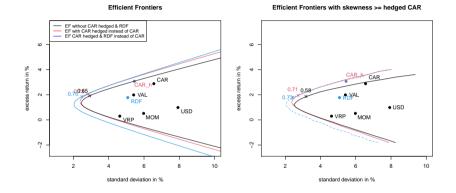
|         | $\lambda_{USD}$ | $\lambda_{CAR}$ | $\lambda_{RDF}$ | $\lambda_{CAR_h}$ | b <sub>USD</sub> | b <sub>CAR</sub> | b <sub>RDF</sub> | $b_{CAR_h}$ | $R^2_{adj}$ | RMSE | $\chi^2$ |
|---------|-----------------|-----------------|-----------------|-------------------|------------------|------------------|------------------|-------------|-------------|------|----------|
| Model 1 | 0.77            | 2.9             |                 |                   | -0.18            | 0.67*            |                  |             | 71.42       | 0.53 |          |
|         | (3.49)          | (2.89)          |                 |                   | (0.31)           | (0.37)           |                  |             |             |      | 93.19%   |
| Model 2 | 0.83            |                 | 1.57*           |                   | 0.34             |                  | 0.75*            |             | 41.97       | 0.67 |          |
|         | (1.28)          |                 | (0.71)          |                   | (0.31)           |                  | (0.45)           |             |             |      | 91.65%   |
| Model 3 | 0.82            |                 |                 | 2.99*             | 0.06             |                  |                  | 0.83*       | 77.68       | 0.5  |          |
|         | (2.19)          |                 |                 | (1.5)             | (0.27)           |                  |                  | (0.39)      |             |      | 96.52%   |

# Asset pricing models with RDF and $CAR_{hedged}$ II



mean return

# Ex-post Efficient Frontiers I



The graphs show the ex-post mean-variance efficient frontiers for various combinations of currency factors. The lines of the right graph represent efficient frontiers restricted to a skewness coefficient of  $> CAR_{hedged}$  (simulated weights). When skewness is considered, the economic value added (in terms of Sharpe ratio) is greater than in the unrestricted, simple mean-variance case.

# Ex-post Efficient Frontiers II

### Weights of tangency portfolios:

| Tangency por          | tfolios                    |             |                        | Tangency pe           | ortfolios skewness         | $s \ge 0.1$ (CAF | Rhedged)               |
|-----------------------|----------------------------|-------------|------------------------|-----------------------|----------------------------|------------------|------------------------|
|                       | CAR <sub>hedged</sub> swap | old factors | all factors except CAR |                       | CAR <sub>hedged</sub> swap | old factors      | all factors except CAR |
| CAR <sub>hedged</sub> | 0.43                       |             | 0.22                   | CAR <sub>hedged</sub> | 0.43                       |                  | 0.16                   |
| RDF                   |                            |             | 0.22                   | RDF                   |                            |                  | 0.29                   |
| CAR                   |                            | 0.43        |                        | CAR                   |                            | 0.54             |                        |
| MOM                   | 0.21                       | 0.25        | 0.16                   | MOM                   | 0.18                       | 0.13             | 0.13                   |
| VAL                   | 0.29                       | 0.32        | 0.30                   | VAL                   | 0.25                       | 0.20             | 0.32                   |
| VRP                   | 0.04                       | 0.12        | 0.00                   | VRP                   | 0.09                       | 0.35             | -0.02                  |
| USD                   | 0.04                       | -0.11       | 0.10                   | USD                   | 0.04                       | -0.22            | 0.12                   |
| Sharpe ratio          | 0.73                       | 0.65        | 0.75                   |                       | 0.71                       | 0.58             | 0.73                   |

- Adding new portfolios yields slight improvement in terms of mean-variance considerations but major value when adding skewness to the consideration.
- CAR<sub>hedged</sub> is responsible for majority of increase in Sharpe ratio.

### Robustness

- Robust to different samples of FX:
  - Resampling many different combinations of 24 FX shows that the result from the whole sample of 30 is not an outlier • graph
- Robust to variation in weighting methodology:
  - Only taking top and bottom 3 (=deciles) currencies according to rank-difference and equal weighting them or
  - Setting all weights with |w| < 0.05 to 0 both show similar results.  $\bigcirc$  graph
- Variation of signal table
  - Implying skewness from different option maturities: 1M is the best, 3M/6M/12M worse but similar.

# Summary

- Option-implied skewness seems to be superior predicting risk than interest rates.
- You can earn the carry risk premium without negative skewness/downside exposure to the equity market. This is good news for explanations of carry which are not crash-risk based.
- You should probably use the crash hedged carry portfolio instead of traditional carry as a factor in AP.
- Also, rank-difference based weighting is a nice solution if you want "corner-portfolios" in a low-dimensional asset class

### References I

- Andrews, D. W. (1991). Heteroskedasticity and autocorrelation consistent covariance matrix estimation. Econometrica: Journal of the Econometric Society, pages 817–858.
- Asness, C. S., Moskowitz, T. J., and Pedersen, L. H. (2013). Value and momentum everywhere. The Journal of Finance, 68(3):929-985.

Brunnermeier, M. K., Nagel, S., and Pedersen, L. H. (2008). Carry trades and currency crashes. NBER macroeconomics annual, 23(1):313-348.

- Burnside, C., Eichenbaum, M., Kleshchelski, I., and Rebelo, S. (2011). Do peso problems explain the returns to the carry trade? The Review of Financial Studies, 24(3):853–891.
- Della Corte, P., Ramadorai, T., and Sarno, L. (2016). Volatility risk premia and exchange rate predictability. Journal of Financial Economics, 120(1):21–40.
- Dobrynskaya, V. (2014). Downside market risk of carry trades. Review of Finance, 18(5):1885-1913.
- Fama, E. F. (1984). Forward and spot exchange rates. Journal of monetary economics, 14(3):319-338.
- Farhi, E., Fraiberger, S. P., Gabaix, X., Ranciere, R., and Verdelhan, A. (2009). Crash risk in currency markets. Technical report, National Bureau of Economic Research.
- Harvey, C. R. and Siddique, A. (2000). Conditional skewness in asset pricing tests. The Journal of finance, 55(3):1263-1295.
- Jurek, J. W. (2014). Crash-neutral currency carry trades. Journal of Financial Economics, 113(3):325-347.
- Kraus, A. and Litzenberger, R. H. (1976). Skewness preference and the valuation of risk assets. The Journal of finance, 31(4):1085–1100.
- Lettau, M., Maggiori, M., and Weber, M. (2014). Conditional risk premia in currency markets and other asset classes. Journal of Financial Economics, 114(2):197–225.
- Lustig, H., Roussanov, N., and Verdelhan, A. (2011). Common risk factors in currency markets. The Review of Financial Studies, 24(11):3731-3777.
- Menkhoff, L., Sarno, L., Schmeling, M., and Schrimpf, A. (2012). Currency momentum strategies. Journal of Financial Economics, 106(3):660-684.
- Newey, W. K. and West, K. D. (1987). A Simple, Positive Semi-definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix. *Econometrica*, 55(3):703–708.
- Schneider, P. and Trojani, F. (2015). Fear trading. Swiss Finance Institute Research Paper, (15-03).
- Schneider, P., Wagner, C., and Zechner, J. (2020). Low-risk anomalies? The Journal of finance, 75(5):2673-2718.

Thank you for your attention!

# summary stats quartile-pfs • back

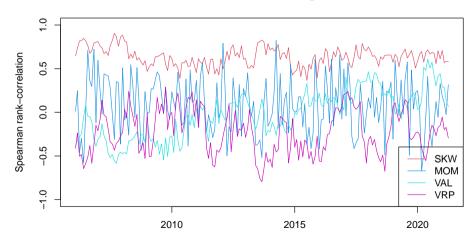
|              | CAR1   | CAR2   | CAR3   | CAR4   | CARhml | SKW1   | SKW2   | SKW3   | SKW4   | SKWhml |
|--------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| mean         | -1.69  | -0.25  | -0.44  | 1.50   | 3.22   | -0.80  | 0.31   | -0.67  | 0.30   | 1.25   |
| sd           | 6.78   | 7.80   | 8.44   | 10.43  | 7.31   | 6.01   | 7.31   | 8.58   | 11.57  | 8.23   |
| skew         | -0.04  | -0.07  | -0.23  | -0.18  | -0.13  | 0.03   | -0.08  | -0.21  | -0.18  | -0.17  |
| kurt         | 0.04   | 0.05   | 0.20   | 0.10   | 0.07   | 0.13   | 0.07   | 0.19   | 0.12   | 0.14   |
| SR           | -0.25  | -0.03  | -0.05  | 0.14   | 0.44   | -0.13  | 0.04   | -0.08  | 0.03   | 0.15   |
| HS2000       | -0.01  | 0.03   | -0.09  | -0.10  | -0.09  | -0.06  | 0.00   | -0.03  | -0.09  | -0.09  |
| KL1976       | -0.03  | 0.08   | -0.23  | -0.33  | -0.24  | -0.13  | 0.00   | -0.10  | -0.31  | -0.12  |
| COV(Rm^2,Ri) | -10.15 | -13.02 | -20.08 | -24.12 | -13.09 | -10.16 | -13.50 | -16.80 | -27.08 | -15.79 |

Skewness formula of Schneider and Trojani (2015)

$$SKEW_{t,T}^{\mathbb{Q}} = \frac{6}{p_{t,T}} \int_{F_{t,T}}^{\infty} \log(\frac{K}{F_{t,T}}) \frac{\sqrt{\frac{K}{F_{t,T}}} C_{t,T}(K)}{K^2} dK - \frac{6}{p_{t,T}} \int_{0}^{F_{t,T}} \log(\frac{F_{t,T}}{K}) \frac{\sqrt{\frac{K}{F_{t,T}}} P_{t,T}(K)}{K^2} dK.$$

- Not standardized = better single measure for tail-risk. Most empirical work focusses on skewness coefficients or risk-reversals. Both are imperfect measures of disaster/tail-risk as they disregard volatility.
- Upper and lower skewness can be separated.
- Suits application to FX: far out of the money options are weighted very small and these are extrapolated in my case (only 10-delta options are actually quoted)

# Similarity of investment signals



rank correlation of investment signals with CAR



# Example weighting RDF •••••

| FX  | CAR   | CARrank | SKEW * 10^5 | SKEWrank | rankdif | weights |
|-----|-------|---------|-------------|----------|---------|---------|
| TWD | -7.08 | 1       | -0.11       | 5        | -4      | -0.046  |
| CHF | -1.96 | 2       | 0.16        | 2        | 0       | 0.000   |
| JPY | -1.93 | 3       | 0.83        | 1        | 2       | 0.023   |
| EUR | -1.48 | 4       | -0.05       | 3        | 1       | 0.011   |
| ILS | -1.26 | 5       | -0.50       | 10       | -5      | -0.057  |
| KRW | -1.18 | 6       | -1.45       | 20       | -14     | -0.161  |
| HUF | -0.96 | 7       | -1.05       | 19       | -12     | -0.138  |
| SEK | -0.92 | 8       | -0.68       | 15       | -7      | -0.080  |
| CLP | -0.91 | 9       | -0.36       | 9        | 0       | 0.000   |
| GBP | -0.84 | 10      | -0.65       | 13       | -3      | -0.034  |
| SGD | -0.59 | 11      | -0.08       | 4        | 7       | 0.080   |
| CAD | -0.43 | 12      | -0.56       | 11       | 1       | 0.011   |
| CZK | -0.31 | 13      | -0.96       | 17       | -4      | -0.046  |
| NOK | -0.26 | 14      | -3.36       | 25       | -11     | -0.126  |
| MYR | -0.21 | 15      | -0.99       | 18       | -3      | -0.034  |
| AUD | -0.20 | 16      | -2.43       | 23       | -7      | -0.080  |
| THB | 0.23  | 17      | -0.26       | 7        | 10      | 0.115   |
| NZD | 0.30  | 18      | -2.24       | 22       | -4      | -0.046  |
| PLN | 0.32  | 19      | -0.81       | 16       | 3       | 0.034   |
| PEN | 1.57  | 20      | -0.30       | 8        | 12      | 0.138   |
| BRL | 2.34  | 21      | -5.02       | 26       | -5      | -0.057  |
| COP | 2.51  | 22      | -1.56       | 21       | 1       | 0.011   |
| RON | 3.37  | 23      | -0.66       | 14       | 9       | 0.103   |
| MXN | 5.56  | 24      | -15.75      | 30       | -6      | -0.069  |
| ZAR | 5.80  | 25      | -3.36       | 24       | 1       | 0.011   |
| RUB | 6.51  | 26      | -6.29       | 28       | -2      | -0.023  |
| IDR | 7.82  | 27      | -5.32       | 27       | 0       | 0.000   |
| PHP | 10.42 | 28      | -0.13       | 6        | 22      | 0.253   |
| TRY | 11.02 | 29      | -6.69       | 29       | 0       | 0.000   |
| INR | 12.67 | 30      | -0.60       | 12       | 18      | 0.207   |

# Unconditional double-sort • back

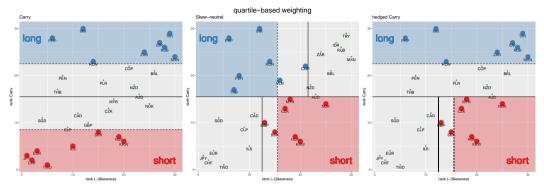


Figure: Illustration of weighting of different portfolios (March 2020). Left graph shows the quartile-based Carry *CAR* portfolio with equal weights. The middle graph shows the skew-neutral porfolio based an (un)conditional double-sort (unconditional is the colored area, conditional are the coloured dots/currencies) and the right graph illustrates the hedged carry, which is compromised of the long leg of *CAR* and short leg of the skew-neutral portfolio. Blue dots represent positive an red dots negative (equal) weights.

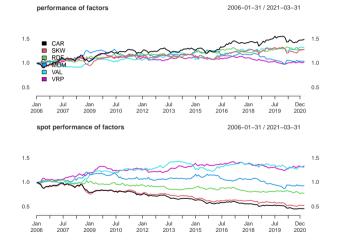
# Unconditional double-sort stats • back

|                       | CARlong | CAR <sub>short</sub> | SNlong | SN <sub>short</sub> | CAR    | SN    | CAR <sub>hedged</sub> |
|-----------------------|---------|----------------------|--------|---------------------|--------|-------|-----------------------|
| mean                  | 2.11    | -0.96                | 1.09   | -0.02               | 3.13   | 1.66  | 2.92                  |
| sd                    | 10.60   | 6.81                 | 7.60   | 10.25               | 7.37   | 6.68  | 7.14                  |
| SR                    | 0.20    | -0.14                | 0.14   | -0.00               | 0.42   | 0.25  | 0.41                  |
| skew                  | -0.18   | -0.04                | -0.14  | 0.06                | -0.13  | 0.14  | -0.02                 |
| kurt                  | 0.11    | 0.04                 | 0.16   | 0.06                | 0.07   | 0.17  | 0.03                  |
| HS2000                | -0.03   | 0.03                 | -0.15  | -0.04               | -0.15  | -0.12 | 0.02                  |
| KL1976                | -0.08   | 0.06                 | -0.38  | -0.14               | -0.46  | -0.30 | 0.03                  |
| $COV(R_m^2, R_i)$     | -17.19  | -7.28                | -18.80 | -19.69              | -18.78 | -0.24 | 1.95                  |
| CARlong               | 1.00    | 0.72                 | 0.84   | -0.76               | 0.77   | -0.22 | 0.37                  |
| CAR <sub>short</sub>  | 0.72    | 1.00                 | 0.77   | -0.90               | 0.11   | -0.50 | -0.23                 |
| SN <sub>long</sub>    | 0.84    | 0.77                 | 1.00   | -0.76               | 0.49   | -0.04 | 0.15                  |
| SN <sub>short</sub>   | -0.76   | -0.90                | -0.76  | 1.00                | -0.26  | 0.68  | 0.32                  |
| CAR                   | 0.77    | 0.11                 | 0.49   | -0.26               | 1.00   | 0.14  | 0.74                  |
| SN                    | -0.22   | -0.50                | -0.04  | 0.68                | 0.14   | 1.00  | 0.65                  |
| CAR <sub>hedged</sub> | 0.37    | -0.23                | 0.15   | 0.32                | 0.74   | 0.65  | 1.00                  |

# Weight statisics • back

| FX  | RDF mean | RDF sd | RDF % > 0 | RDF % < 0 | CAR mean | CAR sd | CAR % > 0 | CAR % < 0 |
|-----|----------|--------|-----------|-----------|----------|--------|-----------|-----------|
| CZK | -12.9    | 7.5    | 6         | 92        | -7.6     | 4.1    | 5         | 95        |
| HUF | -8.7     | 6.8    | 3         | 91        | 1.5      | 7.0    | 58        | 42        |
| SEK | -8.3     | 5.9    | 10        | 90        | -7.8     | 3.4    | 1         | 99        |
| PLN | -7.9     | 6.4    | 6         | 91        | 0.3      | 3.1    | 48        | 46        |
| COP | -5.1     | 6.7    | 14        | 78        | 4.8      | 5.8    | 83        | 17        |
| CLP | -4.1     | 8.9    | 37        | 57        | 2.1      | 5.8    | 74        | 23        |
| KRW | -3.1     | 9.4    | 37        | 56        | -2.2     | 3.8    | 29        | 69        |
| CHF | -2.8     | 4.9    | 21        | 63        | -12.0    | 1.3    | 0         | 100       |
| NOK | -2.8     | 7.2    | 28        | 66        | -2.7     | 3.0    | 19        | 80        |
| RON | -2.5     | 9.6    | 37        | 57        | 3.6      | 6.1    | 73        | 26        |
| ILS | -2.5     | 6.7    | 32        | 62        | -5.6     | 3.6    | 7         | 93        |
| ZAR | -1.9     | 3.0    | 22        | 68        | 10.1     | 1.8    | 100       | 0         |
| MYR | -0.8     | 9.5    | 46        | 48        | -0.8     | 6.2    | 50        | 50        |
| PEN | -0.5     | 8.1    | 44        | 49        | 3.3      | 5.3    | 78        | 22        |
| TWD | -0.2     | 7.1    | 32        | 59        | -9.5     | 5.4    | 9         | 90        |
| EUR | 0.1      | 5.5    | 47        | 40        | -8.7     | 2.5    | 0         | 100       |
| BRL | 0.5      | 5.1    | 54        | 39        | 9.9      | 2.9    | 99        | 1         |
| MXN | 1.8      | 5.9    | 58        | 29        | 7.0      | 3.5    | 100       | 0         |
| JPY | 2.3      | 5.5    | 81        | 9         | -10.8    | 2.3    | 0         | 100       |
| AUD | 3.3      | 7.2    | 72        | 25        | 2.5      | 3.9    | 81        | 18        |
| CAD | 3.3      | 4.9    | 70        | 18        | -4.3     | 2.4    | 3         | 97        |
| GBP | 3.3      | 7.4    | 71        | 23        | -5.1     | 3.3    | 13        | 84        |
| IDR | 3.8      | 7.8    | 69        | 22        | 7.2      | 5.7    | 88        | 11        |
| RUB | 3.9      | 7.4    | 60        | 25        | 8.1      | 5.4    | 89        | 10        |
| NZD | 4.4      | 5.2    | 78        | 16        | 3.4      | 3.9    | 82        | 16        |
| THB | 4.6      | 7.8    | 72        | 26        | 1.1      | 5.1    | 53        | 46        |
| TRY | 5.0      | 5.8    | 68        | 3         | 12.0     | 2.2    | 100       | 0         |
| SGD | 5.5      | 6.9    | 76        | 18        | -5.3     | 4.1    | 11        | 87        |
| PHP | 7.9      | 10.0   | 77        | 21        | 0.5      | 7.1    | 64        | 36        |
| INR | 16.0     | 7.8    | 96        | 4         | 8.6      | 4.5    | 97        | 3         |

### Performances of factors **Dack**



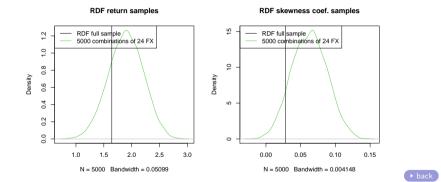
# Robustness to signal (option-maturity)

RDF returns are a bit worse if longer tenors are used for skewness calculations but longer tenors seem to hold similar information):

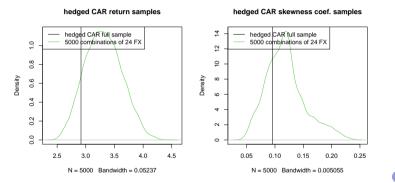
|              | 1-month | 3-month | 6-month | 1-year |
|--------------|---------|---------|---------|--------|
| mean         | 1.643   | 1.280   | 1.069   | 1.151  |
| sd           | 5.067   | 5.383   | 5.157   | 5.319  |
| Sharpe Ratio | 0.324   | 0.238   | 0.207   | 0.216  |
| skew         | 0.028   | 0.106   | 0.041   | 0.114  |
| kurt         | 0.053   | 0.139   | 0.071   | 0.187  |

back

### Robustness to variation in FX-sample: RDF

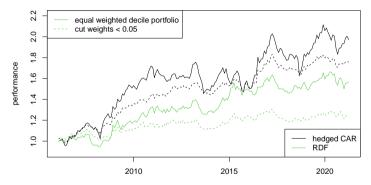


### Robustness to variation in FX-sample: hedged CAR





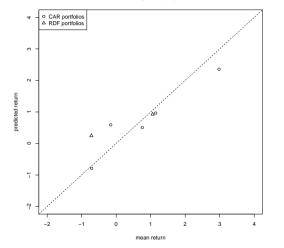
# Robustness in varying weighting methodology •••••



date

|          | RDF cut $ w  < 0.05$ | hedged CAR cut $ w  < 0.05$ | RDF ew decile pf | hedged CAR ew decile pf |
|----------|----------------------|-----------------------------|------------------|-------------------------|
| mean     | 1.41                 | 3.57                        | 2.95             | 4.47                    |
| sd       | 5.59                 | 5.68                        | 7.38             | 9.61                    |
| SR       | 0.25                 | 0.63                        | 0.40             | 0.47                    |
| skewness | 0.03                 | 0.14                        | 0.02             | 0.01                    |
| kurtosis | 0.06                 | 0.14                        | 0.06             | 0.09                    |

# $RDF_{short}$ also outlier in model of Lustig et al. (2011)



AP-model with change in equity VOL from LRV

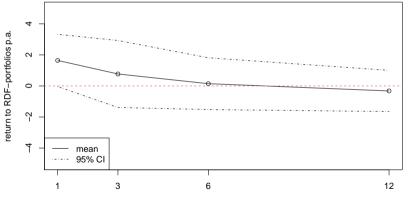


# Further properties of RDF

### RDF only works on 1M-horizon Praph

- The strategy does not offer significant returns when the holding period is extended from 1 month to multiple.
- Return contribution is roughly equal parts short and long leg
  - However, they really only work as a team! graph & table
- There seems to be time variation in the risk premium
  - Returns following months that see an increase in similarity between CAR and SKW have higher returns than months with decreases in similarity. regression & graph

# Risk Premium only on 1-month horizon



holding period in months



# Performances of long and short legs • back • back 2

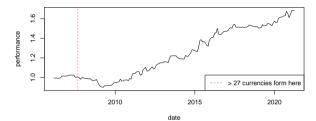


Performances of long & short components

|                      | CARlong | CAR <sub>short</sub> | RDF <sub>long</sub> | RDF <sub>short</sub> | CAR    | RDF   | CAR <sub>hedged</sub> |
|----------------------|---------|----------------------|---------------------|----------------------|--------|-------|-----------------------|
| mean                 | 1.84    | -0.76                | 0.81                | -1.18                | 2.66   | 1.64  | 2.92                  |
| sd                   | 10.16   | 6.80                 | 7.11                | 9.66                 | 6.57   | 5.07  | 5.42                  |
| SR                   | 0.18    | -0.11                | 0.11                | -0.12                | 0.40   | 0.32  | 0.54                  |
| skew                 | -0.20   | -0.03                | -0.21               | -0.15                | -0.15  | 0.03  | 0.10                  |
| kurt                 | 0.11    | 0.04                 | 0.20                | 0.11                 | 0.08   | 0.05  | 0.11                  |
| HS2000               | -0.08   | -0.05                | -0.16               | -0.06                | -0.11  | -0.11 | -0.03                 |
| KL1976               | -0.25   | -0.12                | -0.36               | -0.19                | -0.25  | -0.22 | -0.06                 |
| COV.Rm.2.Ri.         | -22.42  | 8.18                 | -18.25              | -20.46               | -12.33 | 1.36  | -1.88                 |
| CARlong              | 1.00    | 0.77                 | 0.92                | 0.85                 | 0.75   | -0.33 | 0.36                  |
| CARshort             | 0.77    | 1.00                 | 0.85                | 0.94                 | 0.14   | -0.59 | -0.23                 |
| RDF <sub>long</sub>  | 0.92    | 0.85                 | 1.00                | 0.86                 | 0.53   | -0.23 | 0.19                  |
| RDF <sub>short</sub> | 0.85    | 0.94                 | 0.86                | 1.00                 | 0.34   | -0.69 | -0.19                 |
| CAR                  | 0.75    | 0.14                 | 0.53                | 0.34                 | 1.00   | 0.10  | 0.79                  |
| RDF                  | -0.33   | -0.59                | -0.23               | -0.69                | 0.10   | 1.00  | 0.62                  |
| CARhedged            | 0.36    | -0.23                | 0.19                | -0.19                | 0.79   | 0.62  | 1.00                  |



#### weights proportional to change in r-correlation



Greater returns and better Sharpe ratio by weighing returns proportional to the changes in rank-correlation between CAR and SKW.

Explanation: If similarity increases, the signal for those currencies that are dissimilar is stronger?

|                       | RDF                  |      |         |                |                | RDF post 2007        |      |                        |
|-----------------------|----------------------|------|---------|----------------|----------------|----------------------|------|------------------------|
| (Intercept)           | 0.00                 | tir  | ned RDF | (Int           | ercept)        | 0.00                 |      | timed RDF post 2007/10 |
| a such                | (0.00)<br>0.03**     | mean | 3.44    |                | L              | (0.00)<br>0.04***    | mean | 4.95                   |
| corch                 | (0.01)               | sd   | 4.91    | cord           | corch          | (0.01)               | sd   | 5.09                   |
| R <sup>2</sup>        | 0.05                 | SR   | 0.70    | R <sup>2</sup> |                | 0.09                 | SR   | 0.97                   |
| Adj. R <sup>2</sup>   | 0.04                 | skew | 0.17    | Adj            | R <sup>2</sup> | 0.08                 | skew | 0.18                   |
| Num. obs.             | 182                  | kurt | 0.19    | Nur            | n. obs.        | 147                  | kurt | 0.16                   |
| **** p < 0.001; *** p | p < 0.01; * p < 0.05 |      |         | **** p         | < 0.001; **/   | p < 0.01; * p < 0.05 |      |                        |