Relative Entropy and Market Price of Risk during COVID-19

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Motivation(I)

Analyze how the pandemic's financial impact was anticipated by equity option market.

Allows us to gain insights into questions such as:

- when the market started to react,
- how severe the economic impact was estimated to be,
- how these estimates changed as the pandemic unfolded in its first wave.

Motivation(II)

Translate the information extracted from the equity option market to the real world.

- If market participants were risk-neutral, risk-neutral and physical propabilities would coincide.
- Prices in financial markets, however, are in line with risk aversion, not risk neutrality.
- This implies that physical and risk-neutral distributions differ: risk-neutral probability = physical probability x risk aversion adjustment.

Related Literature(I)

- Reactions at the individual stock level and differences in the cross-section (mostly historical studies):
  Albuquerque et al. (2020), Bretscher et al. (2020), Cejnek et al. (2020), Ding et al. (2020), Hassan et al. (2020), Pagano et al. (2020), Ramelli and Wagner (2020).
- Reactions at the aggregate or market level (some rely on forward-looking information):
  Cejnek et al. (2020), Croce et al. (2020), Gerding et al. (2020), Gormsen and Koijen (2020), Ru et al. (2020), Jackwerth (2020).

Related Literature(II)

- Using historical data to construct physical return probabilities: Jackwerth(2000), Ait-Sahalia and Lo (2000), Rosenberg and Engle (2002), Barone-Adesi et al. (2017), Baltussen et al. (2018).
- Constructing forward-looking physical probabilities from risk-neutral ones: Ross(2013), Borovicka et al. (2016), Jensen et al. (2019), Jackwerth and Menner(2020).

Data

- Spot values of the S&P 500 ($S_{t,T}$).
- Implied volatility surface of constant-maturity options.
- Maturity: 1M.
- Moneyness ratios: 50% to 150%, (defined as $F_{t,T}/K$, where $F_{t,T}$ is the forward index value and K is the option strike).
- Risk-free interest rates $r_{t,T}$ in the respective currency.
- Financial stress index: Office of Financial Research Financial Stress Index (OFR FSI).
- Timeframe: 2007/12/05 - 2020/08/25.

Methodology (I): risk-neutral density (RND)

The expected value of a call option can be expressed in terms of the risk-neutral density:

$$C = \int_{K}^{\infty} e^{-rT}(S_T - K)q(S_T)dS_T \quad (1)$$

where $q(S_T)$ - is the risk neutral density (RND).

Breeden and Litzenberger (1978) demonstrate that, given a call price function that is continuous, the RND can be related to the price of the European call option as follows:

$$q(S_T) = e^{-r(T-t)} \frac{\partial^2 C(S_t, K, T, t)}{\partial K^2} \bigg|_{K=S_T} \quad (2)$$

For smoothing and curve-fitting purposes, we prefer to use the non-parametric clamped spline approach proposed by Malz (2014) to avoid arbitrage opportunities.

Methodology(II): Kullback-Leibler divergence (KLD)

For continuous probability distributions P (physical) and Q (risk-neutral) defined on the same probability space, the Kullback-Leibler divergence (KLD) of P from Q is defined as

$$\mathrm{KLD}(P \,||\, Q) = \int_{-\infty}^{\infty} p(\omega) \ln \left(\frac{p(\omega)}{q(\omega)} \right) d\omega, \tag{3}$$

where p and q denote the densities of P and Q.

In our approach, we will use a financial stress index as a proxy for the KLD to construct the physical density p from the risk-neutral density q.

Methodology(II): positivity of KLD

To ensure the positivity of the KLD, we apply the following transformation to the financial stress index:

$$\widehat{KLD}_t = \alpha + \beta \frac{I_t - \min\{I\}}{\sigma_t(I)}, \tag{4}$$

where I_t denotes the respective index value at time t, $\sigma_t(I)$ is the historical volatility of the respective index calculated on the time interval $[0, t]$. The coefficients α and β are used to linearly transform (or scale) the standardized index values. The following values for the coefficients are considered: α=0, β=0.01; α=0, β=0.025; α=0.02, β=0.025.

Results: KLD and physical excess return

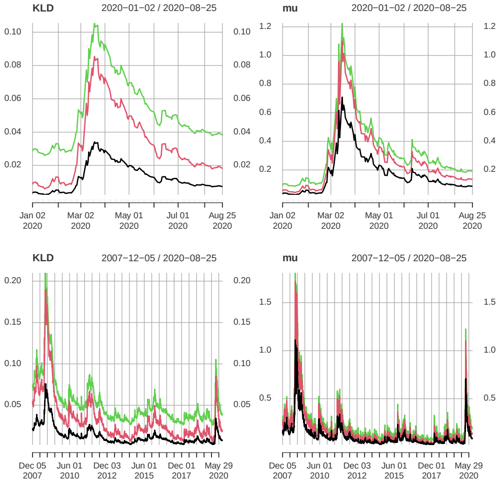

Figure 1: Top panel: outbreak of COVID-19. Bottom panel: whole sample. Black line: α=0, β=0.01. Red line: α=0, β=0.025. Green line: α=0.02, β=0.025.

Results: realized moments vs physical moments

moment	$\alpha = 0$ $\beta = 0.01$	$\alpha = 0$ $\beta = 0.025$	$\alpha = 0.02$ $\beta = 0.025$
realized.exret	0.1786	0.1786	0.1786
median.exret	0.0635	0.0996	0.1390
realized.sd	0.1375	0.1375	0.1375
median.sd	0.1495	0.1455	0.1417
realized.sk	-0.6787	-0.6318	-0.6089
median.sk	-0.6457	-0.6457	-0.6457
m.gamma	3.0126	4.8520	7.2828
m.KLD	0.0081	0.0202	0.0402

Table 1: Estimates obtained from equation (3). As proxies for the unobservable KLD OFR Financial Stress Index (OFR FSI) is used after scaling it according to equation (4).

Results: realized excess return vs physical excess return

$$\text{Realized Moment}_{t,T} = \alpha_1 + \beta_1 E_t[\text{Moment}_{t,T}] + \epsilon_{t,T} \qquad (5)$$

	$\alpha = 0$ $\beta = 0.01$	$\alpha = 0$ $\beta = 0.025$	$\alpha = 0.02$ $\beta = 0.025$
coef	0.1409	0.0908	0.0806
p-value NW	0.0014	0.0014	0.0030
Rsq	0.0411	0.0412	0.0381

Table 2: Estimates obtained from equation (5). The three combinations of scaling parameters are from equation (4).

Conclusion(I)

- Financial markets did not require a significant compensation for bearing COVID-19 pandemic risk until late February.
- From end February and until mid-March the markets reacted strongly with significant daily increases in the physical excess returns.

Preliminary conclusion(II)

- The physical densities show predictive power for the realized moments of empirical return densities.
- The realized moments can not be matched by the physical ones with modest levels of risk aversion.

Thank you for your attention!