

Ex-Ante Risk Factors and Required Structures of the Implied Correlation Matrix

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Presentation Structure

Motivation



Problem Description



Idea of Improvement



Some Empirics

Motivation - Why to solve the Imp.Corr.Matrix

- ▶ Equity Markets are Forward Looking
 - ▶ Forward Looking Asset Pricing requires Forward Looking Expectations. How to estimate?
 1. Either: Ex-Post, and believe that past will repeat itself
 2. Or: Ex-Ante,
 - ▶ Volatility → Option Implied
 - ▶ **Correlation Matrix** → ???
-
- ▶ Forward looking corr. have better predictability, because respond faster to market changes
(e.g., response to COVID-19 in March 2020)

Motivation - Cont.

Research Question:

How to improve the equity implied correlation matrix?

Contribution:

Improvement building on ex-ante specifications

Input Data:

ATM option implied Volatilities, cross-section (S&P500, 2005-2019)

Idea

Factor based asset pricing → requires structure within corr.mat. → clusters

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Some Empirics

Equity's implied correlation Puzzle

The Problem: 1 market with n equities:

- ▶ Expected Volas: $n + 1$ observations (from options)¹
- ▶ Correlation matrix: $\frac{1}{2}(n^2 - n)$ unknowns $> n + 1$

Ok, but why not simply stay with ex-post estimates?

- ▶ Forward-looking → Investor expectations
- ▶ Reacts faster → Higher prediction accuracy for short to mid-term horizons (up to 1 year)

¹Plus sub-indices, if available

CBOE implied Correlation Index

- ▶ starting point for implied corr. matrix
- ▶ same corr.coeff. for all stocks

$$\text{CI : } \sigma_m^2 = w'D \begin{pmatrix} 1 & \bar{c} & \bar{c} & \cdots \\ \bar{c} & 1 & \bar{c} & \cdots \\ \bar{c} & \bar{c} & 1 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} Dw \quad \forall i : D_{ii} = \sigma_i$$

$$\implies \bar{c} = \frac{\sigma_m^2 - w'D^2w}{w'D(1-I)Dw}$$

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Factor Based Asset Pricing

E.g., Fama-French 3 Factor model:

$$\mathbb{E}[r_i] - r_f = \beta_{M,i}MKT + \beta_{S,i}SMB + \beta_{H,i}HML$$

A good and efficient Factor model fulfills:

- ▶ High R^2
- ▶ Significant Coefficients
- ▶ Weak or No Multicollinearity

Hold it!

Factors are built from the Cross-Section,

Weights and Volas given Ex-Ante



Weak/**No Multicollinearity**

if, and only if

Correlation Matrix has certain **structure**

This is what I mean with structure

C.S. sorted on Market Value (SMB Factor)

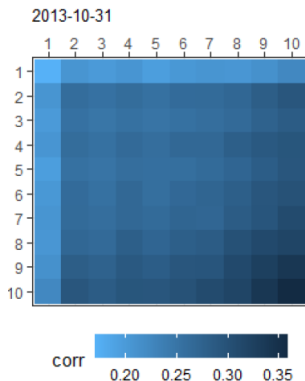


Figure: Ex-Post (1y) Corr.Mat. of S&P500 C.S. sorted on MV; Average corr. per cluster (one cluster = 50 firms)

Let's implement the Structure

- ▶ **CI:** $\sigma_m^2 = w'D\rho Dw$
- ▶ **CII:**² $cov(f, m) = w_f'D\rho Dw \approx 0$

Separate ρ into clusters³ & specify interpolation function \mathcal{F} ,

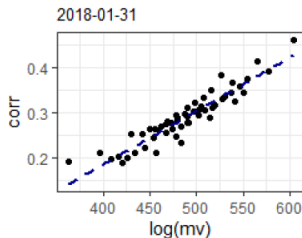
$$\hat{\rho}_{n \times n} := \begin{array}{|c|c|c|} \hline \hat{c}_{1,1} & \dots & \hat{c}_{1,m} \\ \hline \vdots & \ddots & \vdots \\ \hline \hat{c}_{m,1} & \dots & \hat{c}_{m,m} \\ \hline \end{array}$$

²either 0, or some other ex-ante specified value close to 0

³Since factors are portfolios

Mapping Function

1. Grasp a feeling about the relation from Ex-post Pattern:



2. Define Mapping Function & calibrate Ex-Ante:

$$\mathcal{F} : \quad \hat{c}_{i,j} = k \left(\underset{m \times 1}{x_i + x_j} \right) + \underbrace{\phi |x_i - x_j|}_{\text{adjustment term}} + d$$

Motivation



Problem Description

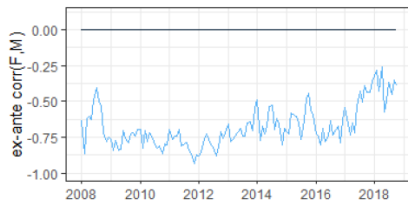


Idea of Improvement

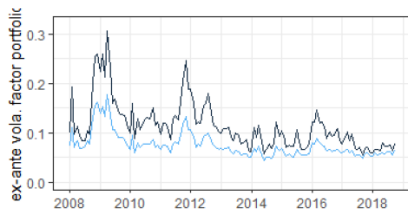


Some Empirics

Empirical Example - 1

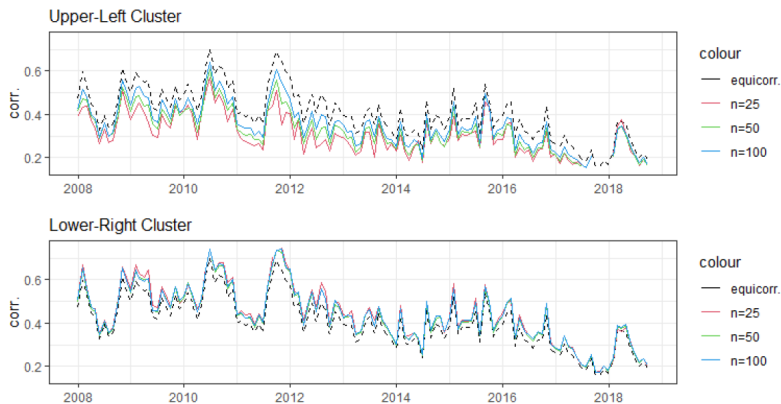


— avg-app. — cluster-app.



— avg-app. — cluster-app.

Empirical Example - 2



Thanks for your attention!

Outlook:

- ▶ Predictability Backtest
- ▶ Calibration for more than two factors

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