Austrian Working Group on Banking and Finance: 35. Workshop

A trip into the Clusterverse

Unsupervised Learning in the Covariance Matrix of Returns

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Introduction: Markowitz(1952) Portfolio Optimization

Target: Maximize the Sharpe Ratio or minimize the Portfolio Variance

Problem: Population not known, need to use sample

Problem with sample: Estimation errors

Result: Poor OOS portfolio performance

$$\min_{\mathbf{w}} \; \mathbf{w}^\top \widehat{\Sigma} \mathbf{w}, \tag{1}$$

$$\text{s.t.} \; \mathbf{w}^\top e = 1, \tag{2}$$

Following DeMiguel et al. (2009)

Clustering in Markowitz Covariance Matrix

What is Clustering and why is it useful?

Clustering is a machine learning procedure to group observations (assets) according to certain similarity measures. It is data driven and granular.

Reducing the asset menu and merging

(e.g. Bjerring et al. (2017), Branger et al. (2019))

Adjust covariances

(e.g. Tola et al. (2008), Panteleo et al. (2011))

Adjusting Corellations

Tola et al. (2008)

- Compare Single Linkage and Average Linkage to Markowitz and RMT optimization.
- Find no superior Sharpe Ratio of any estimator. Main result: The ratio between estimated and realized risk can be decreased, the performance of the estimators strongly depend on the market time.

Pantaelo et al. (2011)

- Compare different market times without considering the Sharpe Ratio (or any performance measure).
- 2 main importances: Ratio between T and N and short sale constraints.
- 10 different estimators (including three clustering algorithms)
- portfolio realized risk, portfolio reliability (i.e. the agreement between realized and predicted risk), and effective portfolio diversification of the portfolios

Research Gaps

Dataset

Tola et al. (2008) use stocks continuosly traded at New York Stock Exchange during the period 1988-1998. Pantaleo et al. (2011) also use stocks traded at New York Stock Exchange, in their case 1997 – 2007.

Common estimator comparison research investigates estimator performance on the Kenneth French Dataset (e.g. DeMiguel et al. (2009), Branger et al. (2017), etc.).

Combination

The different estimators address different targets, while Shrinkage to the identity matrix is data agnostic, Clustering is data driven. Combining both might even further improve the results.

Shrinkage

$$\widehat{\Sigma}_{\mathrm{LW}} = \frac{1}{1+\nu}\widehat{\Sigma} + \frac{\nu}{1+\nu}\widehat{\Sigma}_{\mathrm{target}}, \qquad (4)$$

Following DeMiguel et al. (2009)

Three effects of Shrinkage:

1. Decrease of the covariance -> Leads to stronger diversification, all else equal.
2. Decrease of the variance -> Leads to both stronger diversification and equality of assets.
3. Increase of variance and covariance -> Leads to outlier robustness.

Clustering

Major effect of Clustering:

1. Adjust the Covariance (robustness against outliers).

Clustering is data driven, Shrinkage treats each asset equal.

Introduction

Recap:

1. The minimum variance portfolio tends to outperform the mean-variance portfolio out-of-sample.
2. The sample covariance matrix is still affected by estimation errors, shrinking the sample matrix further improves the performance of the minimum variance portfolio.
3. Hierarchical clustering (on correlations) is neither superior nor inferior to shrinkage on NYSE data (Tola et al. 2008).

Research Gaps:

How do both approaches perform on **sorted portfolios**?

How do **combinations** of different approaches perform? Does shrinkage of correlation clustering estimators improve performance?

Expected Outcomes

Confirm results of Tola et al. (2008).

Confirm results of Pantaleo et al. (2011).

Data

Kenneth French Sorted Portfolios on:

- Investment
- Profitability
- Value
- Market Capitalization
- 12 Industries
- 17 Industries
- 30 Industries
- 48 Industries
- 49 Industries

Methodology

Following DeMiguel et al. (2009):

Rolling window of 120 month estimation window and 1 month investment horizon.

Sample Covariance Estimator

3 Shrinkage Estimators:

Single Factor Shrinkage (Ledoit & Wolf (2003)).

Constant Variance Shrinkage (Ledoit & Wolf (2004a)).

Constant Correlation Shrinkage (Ledoit & Wolf (2004b)).

2 Hierarchical Clustering Estimators:

Weighted Pair-Group Method using arithmetic Averages (WPGMA).

Unweighted Pair-Group Method using arithmetic Averages (UPGMA).

Measurements

Variance (Test statistics following Ledoit & Wolf (2011))

Sharpe Ratio (Test statistics following Ledoit & Wolf (2008))

Turnover

$$(\widehat{\sigma}^i)^2 = \frac{1}{T-\tau-1}\sum_{t=\tau}^{T-1}\left(\mathrm{w}_t^{i^\top} r_{t+1} - \hat{\mu}^i\right)^2,$$

$$\text{with } \hat{\mu}^i = \frac{1}{T-\tau}\sum_{t=\tau}^{T-1}\mathrm{w}_t^{i^\top} r_{t+1}, \qquad (11)$$

$$\widehat{\mathrm{SR}}^i = \frac{\hat{\mu}_i}{\widehat{\sigma}_i}, \qquad (12)$$

$$\text{Turnover} = \frac{1}{T-\tau-1}\sum_{t=\tau}^{T-1}\sum_{j=1}^{N}\left(|\mathrm{w}_{j,t+1}^i - \mathrm{w}_{j,t^+}^i|\right), \qquad (13)$$

Following DeMiguel et al. (2009)

Hypotheses

Hypothesis 1: Clustering performs similar to Shrinkage (to the identity Matrix) on Major Kenneth French Datasets.

Hypothesis 2: Since both approaches address different targets, a combination of both will further improve the performance.

Hypothesis 3: Under shortsale constraints, no estimator outperforms the Minimum Variance Portfolio regarding portfolio variance.

Results I: Without Shortsale Constraints

		Investment Sorted	Profitability Sorted	HML	SMB	12 Industries	17 Industries	30 Industries	48 Industries	49 Industries
Unconstrained	Measurements (OOS)	1	1	1	1	1	1	1	1	1
Min Var U	Variance	0.001455865	0.001530262	0.00167183	0.00164864	0.00120392	0.00119057	0.00134149	0.00162592	0.00162097
	Sharpe Ratio	0.305266789	0.28532836	0.25755066	0.22763602	0.30204624	0.2626235	0.28099517	0.22256603	0.22883034
	Turnover	1.137332506	1.446973083	1.23756561	1.45898291	0.19662194	0.26707094	0.46106784	0.78841621	0.81264799
		2	2	2	2	2	2	2	2	2
LW SF	Variance	0.001592379	0.001691678	0.00169183	0.00170306	0.00116844	0.00113881	0.00115247	0.00115289	0.00114996
	Sharpe Ratio	0.296491355	0.278755009	0.25691275	0.24148163	0.30881729	0.28393193	0.31158431	0.27449157	0.27636274
	Turnover	0.348480991	0.366727113	0.3504449	0.45935055	0.15536766	0.20719081	0.28681994	0.34631124	0.35152601
		3	3	3	3	3	3	3	3	3
LW CC	Variance	0.001951648	0.002039353	0.00191765	0.00354326	0.00118635	0.0011669	0.0012131	0.00120985	0.00119789
	Sharpe Ratio	0.260961293	0.287576791	0.24477985	0.19093902	0.3049007	0.2931718	0.31088337	0.28537545	0.2831401
	Turnover	0.260626849	0.290478192	0.24995022	0.34078813	0.11775681	0.14923195	0.22705562	0.32970875	0.33371513
		4	4	4	4	4	4	4	4	4
LW CV	Variance	0.00138847	0.001470797	0.00162183	0.00161587	0.00113218	0.001103	0.00115529	0.00119989	0.00119383
	Sharpe Ratio	0.317826967	0.30094027	0.26463364	0.24455964	0.3158938	0.29782232	0.3236454	0.28188648	0.28288878
	Turnover	0.213062623	0.222022542	0.22123869	0.20642883	0.12783988	0.16939403	0.2642751	0.37021391	0.37487691
		5	5	5	5	5	5	5	5	5
Average Linkage	Variance	0.001803285	0.001775452	0.00204116	0.00290065	0.00134261	0.00136702	0.00140076	0.00139831	0.00137124
	Sharpe Ratio	0.287709343	0.306408997	0.23466397	0.22311866	0.30071559	0.30301592	0.31106593	0.3209702	0.32075361
	Turnover	0.604817812	0.684014419	0.57942657	0.8371437	0.14661478	0.20456153	0.27371975	0.34003189	0.34339697
		6	6	6	6	6	6	6	6	6
Weighted Linkage	Variance	0.001756665	0.001728879	0.00203514	0.00246343	0.00130193	0.0013063	0.00129814	0.0013548	0.00131312
	Sharpe Ratio	0.301628189	0.306835938	0.23775536	0.22779734	0.29733689	0.29891432	0.30256082	0.30419787	0.31135961
	Turnover	0.64800621	0.663437379	0.58999221	1.02862435	0.15367891	0.23248226	0.34113133	0.47560083	0.49167831
		7	7	7	7	7	7	7	7	7
Average Linkage Shrinked	Variance	0.001398124	0.001433304	0.00163305	0.00207703	0.00118766	0.00122405	0.00129544	0.00129095	0.00127771
	Sharpe Ratio	0.323116699	0.320353268	0.27045289	0.24408008	0.31765982	0.32052554	0.33086438	0.33681901	0.32879815
	Turnover	0.209319359	0.215294546	0.18985532	0.14689612	0.08480757	0.11882598	0.17215236	0.22856796	0.23377509
		8	8	8	8	8	8	8	8	8
Weighted Linkage Shrinked	Variance	0.001388025	0.001433103	0.00164851	0.00190775	0.00117086	0.00118413	0.0012179	0.00126295	0.00123404
	Sharpe Ratio	0.331902964	0.318823739	0.27142196	0.24554756	0.3152255	0.31622846	0.32024829	0.31982738	0.32257653
	Turnover	0.243555073	0.227263055	0.20057115	0.25567019	0.09305092	0.14362839	0.22386854	0.3482724	0.36948184

Results II: With Shortsale Constraints

		Investment Sorted	Profitability Sorted	HML	SMB	12 Industries	17 Industries	30 Industries	48 Industries	49 Industries
Constrained	Measurements (OOS)	1	1	1	1	1	1	1	1	1
Min Var	Variance	0.0015279	0.00167927	0.00177012	0.00179137	0.00117459	0.00116882	0.00119959	0.00121807	0.00121762
	Sharpe Ratio	0.287897844	0.269642668	0.25793892	0.24819964	0.30761094	0.2908003	0.28859302	0.28209552	0.28259433
	Turnover	0.087001606	0.096670503	0.12290939	0.02691815	0.0526068	0.05704452	0.06957768	0.07470458	0.07465296
		2	2	2	2	2	2	2	2	2
LW SF	Variance	0.001531233	0.001681461	0.00177334	0.00179769	0.00116469	0.0011577	0.0011756	0.00118712	0.00118797
	Sharpe Ratio	0.287715014	0.269399625	0.25756236	0.24781624	0.30845867	0.29644391	0.29525425	0.28911993	0.28903196
	Turnover	0.074256497	0.078679885	0.10141758	0.04028135	0.04837294	0.05054248	0.06089726	0.06509193	0.06515445
		3	3	3	3	3	3	3	3	3
LW CC	Variance	0.001529126	0.001690452	0.00177458	0.00181223	0.00117439	0.0011631	0.00118834	0.0011975	0.00119811
	Sharpe Ratio	0.287816244	0.261470719	0.26005165	0.24561843	0.30466624	0.2974855	0.29572876	0.29188694	0.29172938
	Turnover	0.065698075	0.069010078	0.09281948	0.03353726	0.04807452	0.05098631	0.05994232	0.06742974	0.06810653
		4	4	4	4	4	4	4	4	4
LW CV	Variance	0.001527546	0.001673546	0.00176694	0.0017976	0.00116093	0.00116522	0.00118555	0.0012075	0.00120698
	Sharpe Ratio	0.286029422	0.26671416	0.2611402	0.2476317	0.31167621	0.29720993	0.29881316	0.28777484	0.28821716
	Turnover	0.058657275	0.060149303	0.08732596	0.02919285	0.05014871	0.05105795	0.0637061	0.07172694	0.07185502
		5	5	5	5	5	5	5	5	5
Average Linkage	Variance	0.001538112	0.001684216	0.00173815	0.00183753	0.00116018	0.00116176	0.00118692	0.00119891	0.0012079
	Sharpe Ratio	0.283896406	0.265582779	0.26331096	0.25155779	0.31060125	0.30946235	0.3031464	0.30726876	0.30694036
	Turnover	0.087408864	0.091962578	0.10302256	0.06487565	0.04429973	0.05301643	0.06400604	0.08605125	0.08770613
		6	6	6	6	6	6	6	6	6
Weighted Linkage	Variance	0.001535686	0.001687359	0.00174411	0.00183495	0.00114911	0.00114418	0.00115841	0.00119861	0.00120096
	Sharpe Ratio	0.284794631	0.264723669	0.2620246	0.25092322	0.3100986	0.30705351	0.29798979	0.3014084	0.30302867
	Turnover	0.092641957	0.093195195	0.11017438	0.06319018	0.04823616	0.06598569	0.0961243	0.15464364	0.1596949
		7	7	7	7	7	7	7	7	7
Average Linkage Shrinked	Variance	0.001540376	0.001675039	0.00173258	0.00184998	0.00114586	0.00117099	0.00117537	0.00120488	0.00121414
	Sharpe Ratio	0.281757487	0.262862445	0.2676757	0.24743002	0.31881278	0.31078427	0.31639257	0.3136485	0.31172056
	Turnover	0.060707312	0.052035005	0.05880053	0.03045644	0.03867155	0.04290791	0.05605062	0.07686762	0.07770497
		8	8	8	8	8	8	8	8	8
Weighted Linkage Shrinked	Variance	0.001541503	0.001678449	0.00173609	0.00185017	0.00114235	0.00115948	0.0011661	0.00120984	0.00121459
	Sharpe Ratio	0.282815383	0.261429473	0.26609435	0.2465774	0.31739412	0.30940506	0.30784665	0.30271463	0.30417384
	Turnover	0.062209238	0.057200121	0.06371562	0.038524	0.04305569	0.05569119	0.08411409	0.14522564	0.15116137

Results III: Statistical Tests

	Investment Sorted	Profitability Sorted	HML	SMB	12 Industries	17 Industries	30 Industries	48 Industries	49 Industries
1	[1] 0.016	[1] 0.068	[1] 0.14	[1] 0.421	[1] 0.001	[1] 0.001	[1] 0.001	[1] 0.001	[1] 0.001
	[1] 0.3037	[1] 0.1928	[1] 0.5734	[1] 0.1758	[1] 0.2557	[1] 0.001	[1] 0.003	[1] 0.005	[1] 0.018
2	[1] 0.005	[1] 0.022	[1] 0.021	[1] 0.099	[1] 0.024	[1] 0.036	[1] 0.791	[1] 0.025	[1] 0.028
	[1] 0.3816	[1] 0.4725	[1] 0.3926	[1] 0.9441	[1] 0.3776	[1] 0.0769	[1] 0.0909	[1] 0.1998	[1] 0.2757
3	[1] 0.001	[1] 0.001	[1] 0.004	[1] 0.001	[1] 0.019	[1] 0.044	[1] 0.122	[1] 0.82	[1] 0.916
	[1] 0.1738	[1] 0.985	[1] 0.4545	[1] 0.5714	[1] 0.3656	[1] 0.9001	[1] 0.4136	[1] 0.7453	[1] 0.975
5	[1] 0.001	[1] 0.005	[1] 0.001	[1] 0.001	[1] 0.004	[1] 0.001	[1] 0.007	[1] 0.049	[1] 0.052
	[1] 0.4905	[1] 0.6144	[1] 0.3397	[1] 0.969	[1] 0.7962	[1] 0.5055	[1] 0.9151	[1] 0.0849	[1] 0.0879
6	[1] 0.001	[1] 0.015	[1] 0.001	[1] 0.001	[1] 0.004	[1] 0.005	[1] 0.071	[1] 0.091	[1] 0.157
	[1] 0.8142	[1] 0.6244	[1] 0.4256	[1] 0.994	[1] 0.5954	[1] 0.6563	[1] 0.5115	[1] 0.2847	[1] 0.1698
7	[1] 0.776	[1] 0.315	[1] 0.711	[1] 0.001	[1] 0.235	[1] 0.023	[1] 0.086	[1] 0.28	[1] 0.276
	[1] 0.5534	[1] 0.0609	[1] 0.5594	[1] 0.6753	[1] 0.8032	[1] 0.1329	[1] 0.5315	[1] 0.017	[1] 0.046
8	[1] 0.936	[1] 0.348	[1] 0.441	[1] 0.001	[1] 0.341	[1] 0.083	[1] 0.311	[1] 0.411	[1] 0.554
	[1] 0.1429	[1] 0.1069	[1] 0.4795	[1] 0.6753	[1] 0.9341	[1] 0.1848	[1] 0.963	[1] 0.1079	[1] 0.0859

	Investment Sorted	Profitability Sorted	HML	SMB	12 Industries	17 Industries	30 Industries	48 Industries	49 Industries
1	[1] 0.982	[1] 0.408	[1] 0.841	[1] 0.285	[1] 0.111	[1] 0.805	[1] 0.266	[1] 0.442	[1] 0.446
	[1] 0.4306	[1] 0.1059	[1] 0.2947	[1] 0.8312	[1] 0.1948	[1] 0.2488	[1] 0.03	[1] 0.2817	[1] 0.2847
2	[1] 0.544	[1] 0.311	[1] 0.561	[1] 0.943	[1] 0.745	[1] 0.387	[1] 0.411	[1] 0.126	[1] 0.16
	[1] 0.3257	[1] 0.1399	[1] 0.1638	[1] 0.963	[1] 0.4945	[1] 0.8102	[1] 0.4066	[1] 0.9211	[1] 0.982
3	[1] 0.791	[1] 0.234	[1] 0.493	[1] 0.504	[1] 0.417	[1] 0.774	[1] 0.978	[1] 0.633	[1] 0.699
	[1] 0.3816	[1] 0.1778	[1] 0.7522	[1] 0.8272	[1] 0.3367	[1] 0.956	[1] 0.7003	[1] 0.6623	[1] 0.7143
5	[1] 0.312	[1] 0.461	[1] 0.067	[1] 0.115	[1] 0.835	[1] 0.762	[1] 0.97	[1] 0.721	[1] 0.954
	[1] 0.5095	[1] 0.7942	[1] 0.6973	[1] 0.5105	[1] 0.9071	[1] 0.0909	[1] 0.5794	[1] 0.0539	[1] 0.0689
6	[1] 0.481	[1] 0.369	[1] 0.2	[1] 0.116	[1] 0.374	[1] 0.159	[1] 0.119	[1] 0.736	[1] 0.809
	[1] 0.7343	[1] 0.6434	[1] 0.9291	[1] 0.5455	[1] 0.8332	[1] 0.1708	[1] 0.8382	[1] 0.2338	[1] 0.1439
7	[1] 0.279	[1] 0.936	[1] 0.17	[1] 0.16	[1] 0.366	[1] 0.855	[1] 0.603	[1] 0.865	[1] 0.828
	[1] 0.1778	[1] 0.2687	[1] 0.1309	[1] 0.8671	[1] 0.3217	[1] 0.0529	[1] 0.028	[1] 0.004	[1] 0.015
8	[1] 0.306	[1] 0.787	[1] 0.38	[1] 0.7	[1] 0.299	[1] 0.712	[1] 0.35	[1] 0.957	[1] 0.818
	[1] 0.4286	[1] 0.1439	[1] 0.2507	[1] 0.979	[1] 0.4695	[1] 0.0989	[1] 0.3427	[1] 0.1868	[1] 0.1289

Summary of Results

The constant Variance Shrinkage (Ledoit & Wolf (2004a)) is dominant regarding portfolio variance without shortsale constraints. With shortsale constraint though, no statistical significant difference between variances can be found (confirming Pantaleo et al. (2011)).

Both clustering and shrinkage tend to perform similar after statistical tests regarding Sharpe ratios (confirming Tola et al. (2008)).

The Average Linkage clustered and shrinked portfolio is dominant in portfolio turnover over all datasets.

Shrinking the clustered portfolios improves their performance in most shortsale-unconstrained setups regarding all measurements, with some statistical tests being insignificant).

Even in situations, in which the constant variance shrinkage is inferior to clustering, using the shrinkage on the clustered matrix improves performance.

Why?

Hypothesis 1: The Clustering is better in adjusting Outliers, the shrinkage is directly affected by the clustering because after cluster shrinkage parameters tend to be higher.

Hypothesis 2: The Combined approach is just a weighted average between both single approaches, portfolio weights frequently lay between both estimation allocations.

Hypothesis 3: The Clustering smoothes unpriced risk factors in industries.

The Truth?

Is unclear.. Further research is necessary:

How do shrinkage and clustering interact on different dimensions?

Another target in future research: Compute data-driven counterfactual covariance matrices introducing new feature dimensions (industry, characteristics or beta clustering).

Link between mispricing, unpriced risk and the covariance matrix:

Clustering in the Residual Covariance Matrix (MacKinlay and Pastor (2000))

Clustering in Alpha and Beta Portfolios (Uppal & Zaffaroni (2015))

Thank you for your Attention!

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