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Short-term exuberance and long-term stability: A simultaneous optimization of stock return predictions for short and long horizons*

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Why a simultaneous approach of short- and long-term market forecasting?

- Most research is targeted towards optimizing investments over **(extreme) short time horizons**.
- A pure short-term approach is **not useful for pensioners**. Over a 30-year period, using a fixed mean return and standard deviation might be a pitfall.
- Understanding the movements of the stock market is important for providing long-term savers with **sufficient wealth at retirement**.
- A proper econometric approach to long-term savings needs
  - to be able to **account for both** the long and the short term concurrently
  - aiming to line up the long-term projections while **circumventing inaccurate trading due to short-term bubbles** in the market.

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Our contribution:

- We provide a **general strategy** in support of such a novel econometric model.
- We consider the standard case of returns in excess of the short-term interest rate and in excess of inflation.
- The application of (nonlinear) predictive regressions for two different horizons **reduces individually the noise** for short- and long-term investments.
- By combining (optimal) predictions of different horizons we **reduce further the noise** for the short-term investment.

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Overview:

- The financial model
- The validation criterion
- The econometric model for combined predictions
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- Based on (nominal) **dividends** D_t paid during year t and (nominal) **stock price** P_t at the end of year t, we investigate **stock returns**

$$S_t = \frac{P_t + D_t}{P_{t-1}} \tag{1}$$

- Stock returns in **excess** (log-scale) of a given **benchmark** $B^{(A)}_{t-1}$ for $A \in \{R, C\}$:

$$Y^{(A)}_t = \ln \frac{S_t}{B^{(A)}_{t-1}} \tag{2}$$

- The considered benchmarks are

$$B^{(R)}_t = 1 + \frac{R_t}{100}, \quad B^{(C)}_t = \frac{CPI_t}{CPI_{t-1}} = 1 + \pi_t,$$

for the **short-term interest rate** R_t and the **consumer price index** CPI_t (with inflation rate π_t).

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The one-year horizon

- A predictive **nonparametric** regression model:

$$Y_t^{(A)} = m_{1y}(X_{t-1}^{(A)}) + \xi_t, \tag{3}$$

- The **unknown smooth function** in eq. (3)

$$m_{1y}(x^{(A)}) = \mathbb{E}(Y^{(A)}|X^{(A)} = x^{(A)}), \; x^{(A)} \in \mathbb{R}^q$$

  is estimated with a **local-linear smoother**,

- The errors in eq. (3) follow a **martingale difference process** ξ_t, i.e. they are serially uncorrelated zero-mean random error terms, given the past, of unknown cond. heterosced. form $\sigma(x^{(A)})$.

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Double benchmarking

- Popular (lagged) **predictive variables** for $X_{t-1}^{(A)}$ are used as dividend-by-price $d_{t-1}^{(A)}$, earnings-by-price $e_{t-1}^{(A)}$, short-term interest $r_{t-1}^{(A)}$, long-term interest $l_{t-1}^{(A)}$, inflation $\pi_{t-1}^{(A)}$, term spread $s_{t-1}^{(A)}$.
- Independent and dependent variables adjusted according to the **same benchmark**.
- We use the **transformed predictive variables**:

$$X_{t-1}^{(A)} = \begin{cases} \frac{1+X_{t-1}}{B_{t-1}^{(A)}}, & X \in \{d, e, r, l, \pi\} \\ \frac{s_{t-1}}{B_{t-1}^{(A)}} = \frac{l_{t-1}-r_{t-1}}{B_{t-1}^{(A)}} \end{cases}, \quad A \in \{R, C\}. \tag{4}$$

- **Balanced dimension**: Additional economic structure in prediction without extra cost in terms of increased dimensionality.

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Longer horizons

- For longer horizons T we consider the sum of annual continuously compounded returns:

$$Z_t^{(A)} = \sum_{i=0}^{T-1} Y_{t+i}^{(A)}.$$

- Note that we use here **overlapping returns** $Z_t^{(A)}$, which require a careful econometric modelling.

- Assume a **linear relationship** between $Y_t^{(A)}$ and $X_{t-1}^{(A)}$, as well as **persistence** of the forecasting variable (treating the variables as deviations from their means):

$$Y_t^{(A)} = \beta X_{t-1}^{(A)} + \xi_t \quad \text{and} \quad X_t^{(A)} = \gamma X_{t-1}^{(A)} + \eta_t,$$

with ξ_t as before and η_t being white noise.

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- T-year regression problem implied by this pair of one-year regressions:

$$\begin{aligned} Z_t^{(A)} &= Y_t^{(A)} + \ldots + Y_{t+T-1}^{(A)} \\ &= (\beta X_{t-1}^{(A)} + \xi_t) + \ldots + (\beta X_{t+T-2}^{(A)} + \xi_{t+T-1}) \\ &= \beta \sum_{i=0}^{T-1} \gamma^i X_{t-1}^{(A)} + \beta \sum_{i=0}^{T-1} \sum_{j=0}^{T-1-i} \gamma^j \eta_{t+i} + \sum_{i=0}^{T-1} \xi_{t+i} \\ &= \phi X_{t-1}^{(A)} + \nu_t, \end{aligned}$$

  i.e. a decomposition in a **predictive part** depending on the variable $X_{t-1}^{(A)}$ and an **unpredictable error** term ν_t.

- To avoid **functional misspecification** due to our simplistic assumption, we allow for nonlinearity :

$$Z_t^{(A)} = m_{Ty}(X_{t-1}^{(A)}) + \nu_t, \tag{5}$$

  where the error ν_t is **serially correlated** by construction.

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- As we use a nonparametric technique, we require an **adequate measure of the predictive power**.
- For model as well as optimal bandwidth selection, we use the validated $R^2_{V,ky}$ of Nielsen and Sperlich (2003) based on a **leave-*l*-out cross-validation**:

$$R^2_{V,ky} = 1 - \frac{\sum_t (W_t^{(A)} - \hat{m}_{-t,ky})^2}{\sum_t (W_t^{(A)} - \bar{W}_{-t}^{(A)})^2}, \tag{6}$$

- **Leave-*l*-out estimators** are used:
  - $\hat{m}_{-t,ky}$ for the nonparametric function m_{ky} ($k \in \{1, T\}$) and
  - $\bar{W}_{-t}^{(A)}$ for the unconditional mean of $W_t^{(A)}$ ($W_t^{(A)} \in \{Y_t^{(A)}, Z_t^{(A)}\}$).
- Both are computed by removing l observations around the tth time point.

The validation criterion II

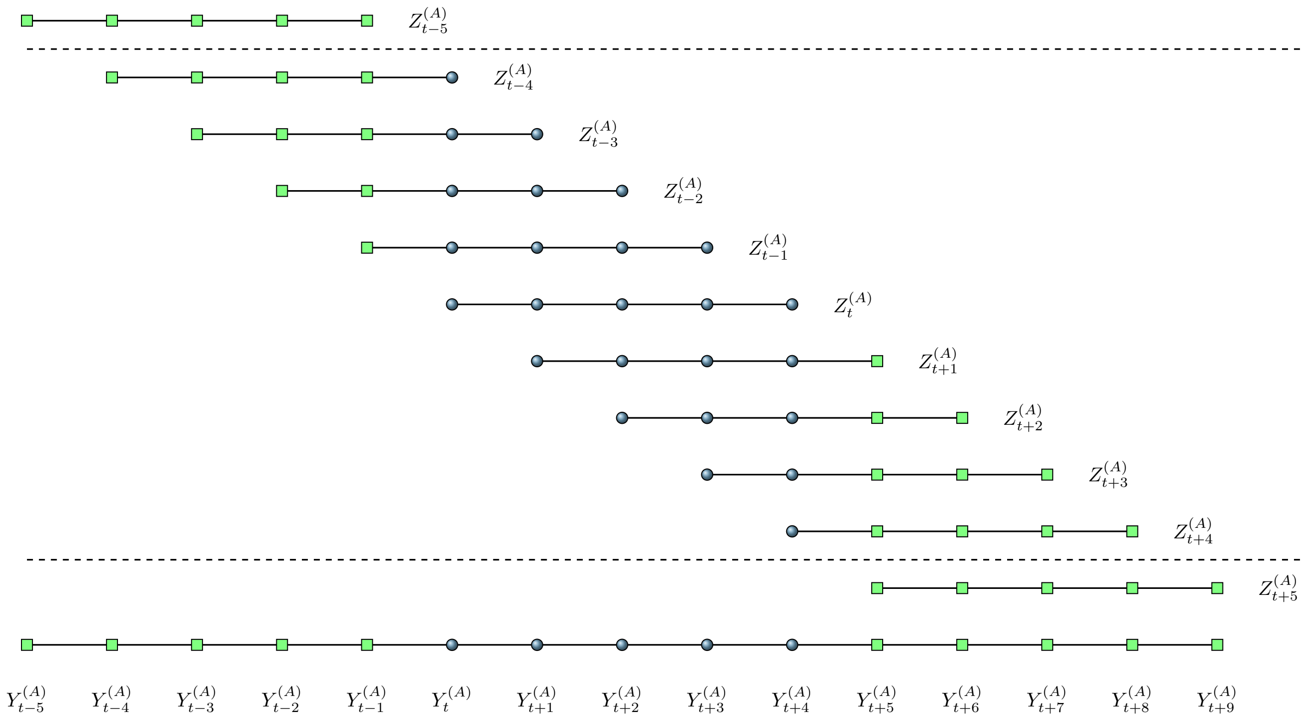

Figure: Illustration of the leave-nine-out set (between the dashed lines) of $Z_{t-4}^{(A)}, \ldots, Z_{t+4}^{(A)}$ which include at least one element of $Y_t^{(A)}, \ldots, Y_{t+4}^{(A)}$ (see bottom).

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Components of our model

- The change in earnings growth as one of the **key drivers** of stock prices P.
- The **autoregressive development** of the earnings variable $e^{(A)}$

$$e_t^{(A)} - e_{t-1}^{(A)} = \rho\,(e_{t-1}^{(A)} - \bar{e}^{(A)}) + \eta_t \quad \Leftrightarrow \quad e_t^{(A)} = \gamma_0 + \gamma_1 e_{t-1}^{(A)} + \eta_t \tag{7}$$

- The **linear predictive model** for the earnings variable $e^{(A)}$

$$Y_{t+1}^{(A)} = \beta_0 + \beta_1 e_t^{(A)} + \xi_{t+1} \tag{8}$$

- With eq. (7) and (8) and the corresponding OLS estimates c_0, c_1, b_0, b_1, which we will keep fixed in the following steps, we can now **forecast out-of-sample**

$$\hat{Y}_{n+1}^{(A)}, \hat{Y}_{n+2}^{(A)}, \ldots, \hat{Y}_{n+T}^{(A)}$$

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- Remember that we want match the optimal $1y$ and Ty predictions $\hat{m}_{1y}$ and $\hat{m}_{Ty}$ from our preferred financial models.
- Therefore, we will **correct** $\hat{Y}^{(A)}_{n+1}, \hat{Y}^{(A)}_{n+2}, \dots, \hat{Y}^{(A)}_{n+T}$ in a simple linear way:

$$\hat{Y}^{(A),c}_{n+1} = \alpha_0 + \alpha_1 \hat{Y}^{(A)}_{n+1} + \varepsilon_{n+1} \tag{9}$$
$$\hat{Y}^{(A),c}_{n+2} = \alpha_0 + \alpha_1 \hat{Y}^{(A)}_{n+2} + \varepsilon_{n+2} \tag{10}$$
$$\vdots$$
$$\hat{Y}^{(A),c}_{n+T} = \alpha_0 + \alpha_1 \hat{Y}^{(A)}_{n+T} + \varepsilon_{n+T}, \tag{11}$$

with unknown parameters α_0 and α_1, independent error terms $\varepsilon_{n+1} \sim \mathcal{N}(0, \sigma_1^2)$ and $\varepsilon_{n+2}, \dots, \varepsilon_{n+T} \sim \mathcal{N}(0, \sigma_2^2)$ with unknown (probably different) variances σ_1^2 and σ_2^2.

- For the corrected T-year return, we get directly

$$Z^{(A),c}_{n+T} = \sum_{k=1}^{T} \hat{Y}^{(A),c}_{n+k} = \alpha_0 T + \alpha_1 \sum_{k=1}^{T} \hat{Y}^{(A)}_{n+k} + \sum_{k=1}^{T} \varepsilon_{n+k}. \tag{12}$$

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- **Calibration** of the 1st and 2nd moments of the corrected one-year and T-year returns

$$\mathbb{E}(\hat{Y}_{n+1}^{(A),c}) = \hat{\mu}_{1y}, \quad \mathbb{E}(\hat{Z}_{n+T}^{(A),c}) = \hat{\mu}_{Ty}, \\ \mathrm{Var}(\hat{Y}_{n+1}^{(A),c}) = \hat{\sigma}^2_{1y}, \quad \mathrm{Var}(\hat{Z}_{n+T}^{(A),c}) = \hat{\sigma}^2_{Ty}, \tag{13}$$

- gives us the model parameters

$$\alpha_0 = \hat{\mu}_{1y} - \alpha_1 \left(b_0 + b_1 e_n^{(A)} \right), \quad \alpha_1 = \frac{\hat{\mu}_{Ty} - T\hat{\mu}_{1y}}{S - b_0 T - b_1 T e_n^{(A)}}, \tag{14}$$

$$\sigma_1^2 = \hat{\sigma}^2_{1y}, \quad \sigma_2^2 = \frac{1}{T-1}\left(\hat{\sigma}^2_{Ty} - \hat{\sigma}^2_{1y} \right) \tag{15}$$

where

$$S := b_0 T + c_0 b_1 \sum_{k=2}^{T} \sum_{i=0}^{k-2} c_1^i + b_1 e_n^{(A)} \sum_{i=0}^{T-1} c_1^i,$$

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- **US stock market data** provided by Robert Shiller, revised and updated over the period **1872–2020**:

Table: US market data (1872–2020).

	Max	Min	Mean	Sd	Skew	Exc. kurt
S&P stock price index	3278.20	3.25	297.62	607.94	2.57	6.62
Dividend accruing to index	58.24	0.18	6.39	11.36	2.52	6.35
Earnings accruing to index	139.47	0.16	14.80	28.17	2.47	5.66
Short-term interest rate	14.93	0.07	3.99	2.49	0.94	2.27
Long-term interest rate	14.59	1.76	4.51	2.25	1.80	3.68
Consumer price index	257.97	6.47	60.32	74.02	1.34	0.35

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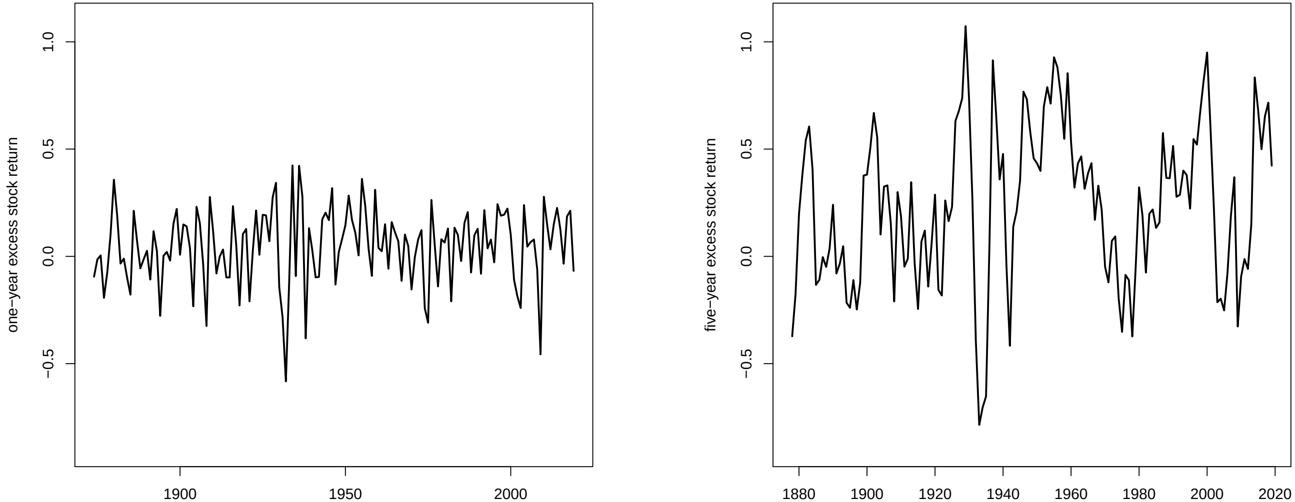

Figure: Left: one-year stock returns in excess of the risk-free benchmark. Right: five-year stock returns in excess of the risk-free benchmark. Period: 1872–2020. Data: annual S&P 500.

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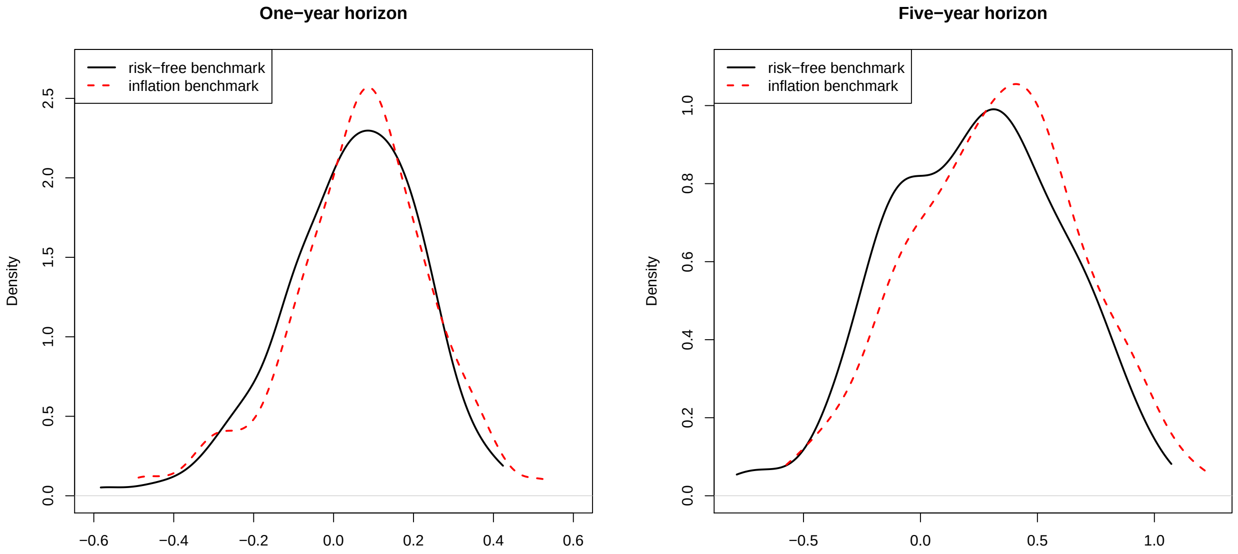

Figure: Kernel density estimates of the probability density function of returns transformed with the risk-free benchmark (solid) and the inflation benchmark (dotted). Left: one-year horizon. Right: five-year horizon. Period: 1872–2020. Data: annual S&P 500.

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Table: Predictive power for one-year excess stock returns: double benchmarking

Benchmark $B^{(A)}$	Explanatory variable(s) $X_{t-1}^{(A)}$					
	$d^{(A)}$	$e^{(A)}$	$r^{(A)}$	$l^{(A)}$	$\pi^{(A)}$	$s^{(A)}$
Short-term rate	3.0	5.0	–	**9.6**	-1.3	**9.6**
Inflation	10.2	**12.0**	7.1	10.4	–	6.6
	$(d^{(A)}, s^{(A)})$	$(e^{(A)}, s^{(A)})$	$(r^{(A)}, s^{(A)})$	$(l^{(A)}, s^{(A)})$	$(\pi^{(A)}, s^{(A)})$	
Short-term rate	9.8	**10.7**	–	–	7.2	
Inflation	15.4	**17.5**	14.7	14.6	–	

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Table: Predictive power for five-year excess stock returns: double benchmarking

Benchmark $B^{(A)}$	Explanatory variable(s) X_{t-1}					
	$d^{(A)}$	$e^{(A)}$	$r^{(A)}$	$l^{(A)}$	$\pi^{(A)}$	$s^{(A)}$
Short-term rate	11.7	10.6	–	**15.9**	-2.4	**15.9**
Inflation	10.8	**12.4**	5.5	8.6	–	1.0
	$(d^{(A)}, s^{(A)})$	$(e^{(A)}, s^{(A)})$	$(r^{(A)}, s^{(A)})$	$(l^{(A)}, s^{(A)})$	$(\pi^{(A)}, s^{(A)})$	
Short-term rate	**22.2**	21.8	–	–	16.3	
Inflation	13.1	**14.9**	10.1	10.2	–	

Empirical results III

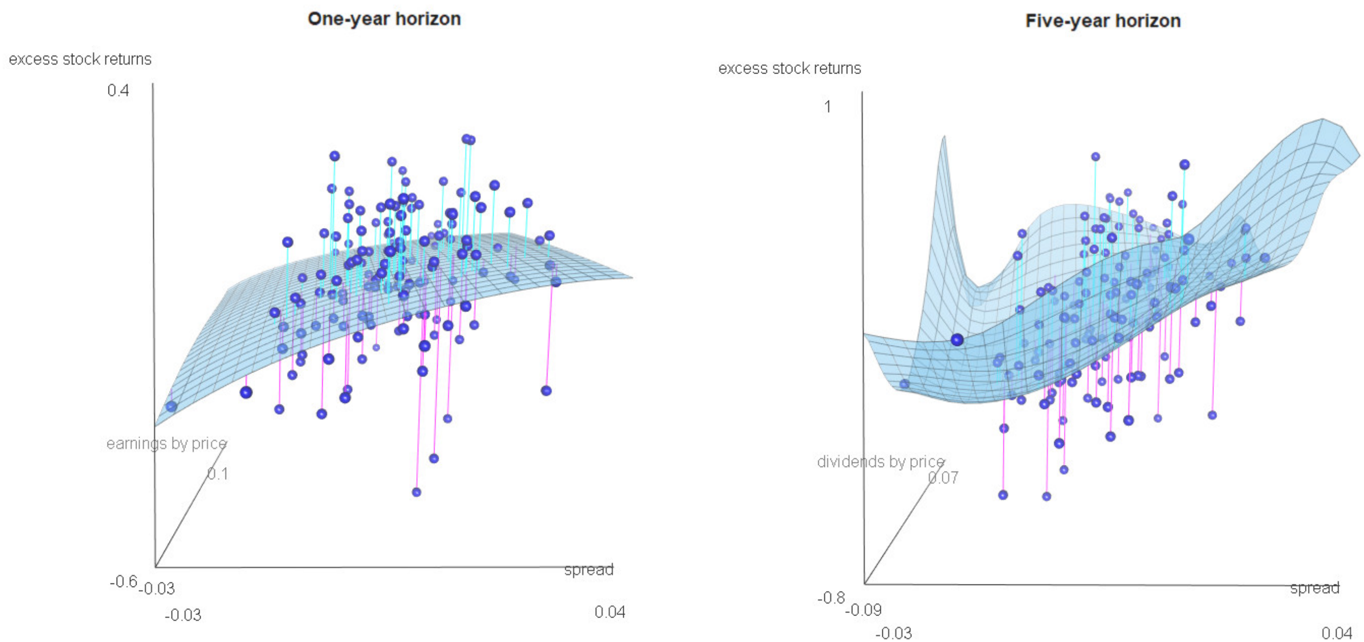

Figure: Double risk-free benchmark. Relation between excess stock returns and predictive variables: the earnings-by-price ratio and the spread, one-year horizon (left), the dividend-by-price ratio and the spread, five-year horizon (right).

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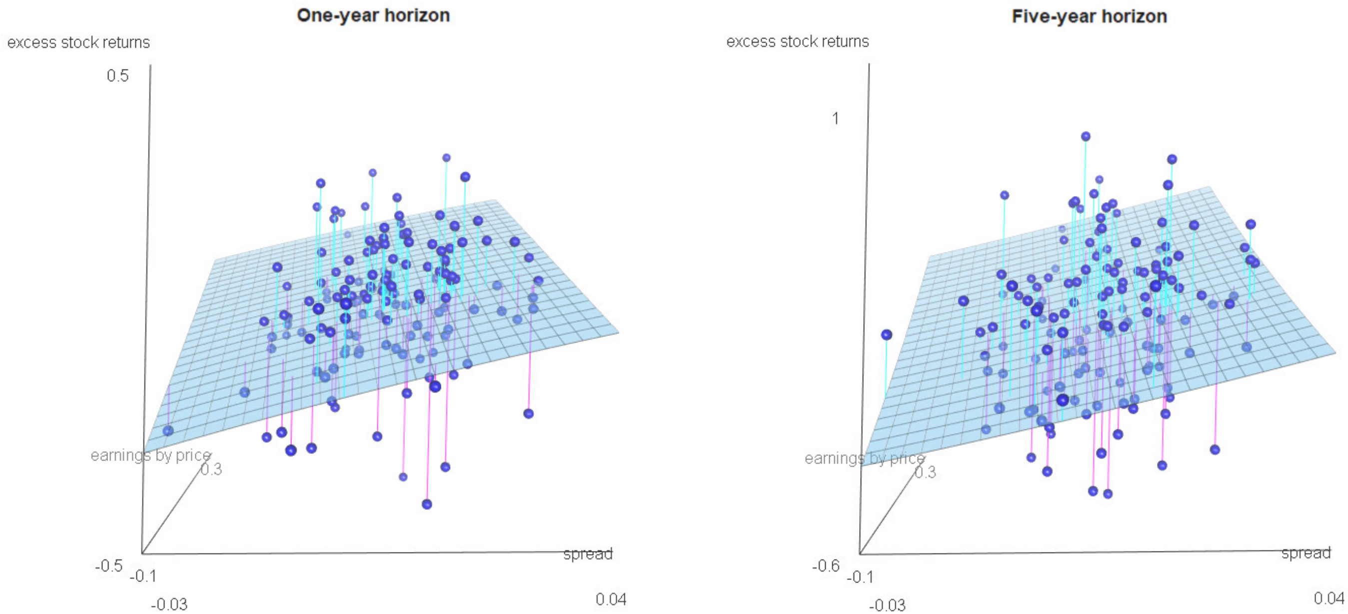

Figure: Double inflation benchmark. Relation between excess stock returns and predictive variables: the earnings-by-price ratio and the spread, one-year horizon (left), the earnings-by-price ratio and the spread, five-year horizon (right).

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Table: Estimated parameters (and standard deviations in brackets) of the linear models (7) and (8) used for the econometric model

Benchmark	short-term interest rate		inflation rate	
Dependent variable	$e^{(R)}_{t+1}$	$Y^{(R)}_{t+1}$	$e^{(C)}_{t+1}$	$Y^{(C)}_{t+1}$
(Intercept)	0.0066**	0.0035	0.0373***	0.0069
	(0.0022)	(0.0201)	(0.0063)	(0.0186)
$e^{(A)}_t$	0.7976***	1.3522**	0.2859***	1.1144***
	(0.0500)	(0.4531)	(0.0799)	(0.2350)
R^2	0.6384	0.0579	0.0817	0.1343
Adj. R^2	0.6359	0.0514	0.0753	0.1283
Num. obs.	146	147	146	147
RMSE	0.0186	0.1683	0.0572	0.1684

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

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Table: Estimated parameters of econometric model: (a) short-term interest rate benchmark, (b) inflation benchmark.

Benchmark	short-term interest rate				inflation rate			
	$\hat{\mu}_{ky}$	$\hat{\sigma}_{ky}$	$R^2_{V,ky}$	σ	$\hat{\mu}_{ky}$	$\hat{\sigma}_{ky}$	$R^2_{V,ky}$	σ
one-year ($k=1$)	4.30	16.34	10.67	17.28	4.15	16.38	17.53	18.04
five-year ($k=5$)	18.81	32.33	22.18	36.65	27.41	33.52	14.85	36.33
	α_0	α_1	σ_1	σ_2	α_0	α_1	σ_1	σ_2
parameter	13.70	-2.30	16.34	13.95	-0.17	0.95	16.38	14.62

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Table: Predicted excess stock returns from econometric model: (a) short-term interest rate benchmark, (b) inflation rate benchmark.

Benchmark	short-term interest rate			inflation rate		
period	$e^{(R)}$	$\hat{Y}^{(R)}$	$\hat{Y}^{(R),c}$	$e^{(C)}$	$\hat{Y}^{(C)}$	$\hat{Y}^{(C),c}$
n	2.76	–	–	3.47	–	–
$n+1$	2.87	4.08	4.30	4.72	4.56	4.15
$n+2$	2.95	4.23	3.97	5.08	5.95	5.47
$n+3$	3.02	4.34	3.71	5.18	6.35	5.85
$n+4$	3.07	4.43	3.50	5.21	6.47	5.95
$n+5$	–	4.50	3.33	–	6.50	5.98

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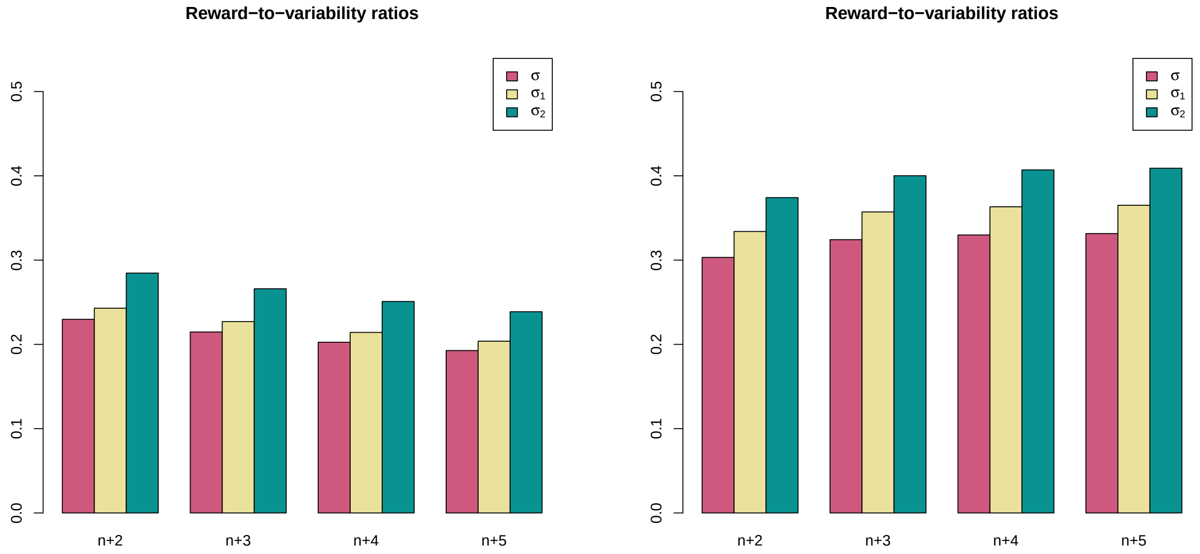

Figure: Comparison of reward-to-variability-ratios based on corrected one-year predictions for periods $n+2, \ldots, n+5$ and σ (red), σ_1 (yellow) and σ_2 (green), Left: the risk-free benchmark. Right: the inflation benchmark.

Empirical results IX

Table: One-year and five-year ahead real-time forecasts based on recent monthly US stock market variables: (a) short-term interest rate benchmark, (b) inflation rate benchmark.

monthly US stock market data							short-term interest rate				inflation rate			
date	P	D	E	R	L	π	$\widehat{RP}$	$\hat{Y}_{nom}$	$\hat{Y}_{real}$	$\hat{Z}^{(R)}$	$\widehat{RP}$	$\hat{Y}_{nom}$	$\hat{Y}_{real}$	$\hat{Z}^{(C)}$
2019-07	2996.11	56.46	134.48	1.91	2.06	1.81	2.99	4.89	3.07	17.17	1.94	3.85	2.06	23.49
2019-08	2897.45	56.84	133.69	1.73	1.63	1.75	2.46	4.18	2.43	14.08	1.38	3.11	1.38	22.36
2019-09	2982.16	57.22	132.90	1.75	1.70	1.71	2.44	4.17	2.46	15.06	1.37	3.12	1.42	22.40
2019-10	2977.68	57.56	135.09	1.57	1.71	1.76	3.42	4.98	3.21	15.63	2.31	3.88	2.14	23.65
2019-11	3104.90	57.90	137.28	1.53	1.81	2.05	3.82	5.33	3.28	13.17	2.66	4.19	2.16	23.59
2019-12	3176.75	58.24	139.47	1.51	1.86	2.29	4.04	5.54	3.25	10.79	2.85	4.36	2.10	23.41
2020-01	3278.20	58.69	131.76	1.49	1.76	2.49	3.37	4.85	2.36	6.82	2.11	3.60	1.15	21.61
2020-02	3277.31	59.13	124.04	1.37	1.50	2.33	2.75	4.11	1.78	6.33	1.49	2.86	0.56	20.56
2020-03	2652.39	59.58	116.33	0.32	0.87	1.54	6.04	6.36	4.82	19.26	4.90	5.22	3.69	26.43
2020-04	2761.98	59.72	110.63	0.18	0.66	0.33	5.57	5.75	5.42	29.99	4.63	4.81	4.48	28.04
2020-05	2919.61	59.87	104.93	0.16	0.67	0.12	5.25	5.41	5.29	31.36	4.32	4.48	4.36	27.78
2020-06	3104.66	60.01	99.23	0.18	0.73	0.65	4.91	5.09	4.45	26.02	3.86	4.04	3.39	25.83
2020-07	3207.62	59.62	99.03†	0.15	0.62	0.99	4.56	4.71	3.73	20.92	3.40	3.55	2.57	24.25
2020-08	3391.71	59.24	98.82†	0.13	0.65	1.31	4.56	4.69	3.38	16.89	3.33	3.46	2.16	23.41
2020-09	3365.52	58.85	98.62†	0.13	0.68	1.37	4.67	4.80	3.43	16.55	3.44	3.57	2.21	23.49

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Summary:

- We combine optimal long-term and short-term information to provide a state-of-the-art econometric model that serves **short-term market timing** as well as **long-term asset-allocation strategy** for the long-term saver.
- We interestingly find a **high short-term (one-year) standard deviation** insinuating presence of bubbles in the returns, consistently with the conclusion of Mase (1999) for short-horizon stock returns.
- This suggests that long-term investors should disregard short-term econometric models when deciding on their long-term asset allocation.
- We conclude that, for a given risk appetite level, the ability of adding equity exposure to result in increased long-term savers' portfolio return is significant as it provides **better pensions for everyone**.

Thank you for your attention!

Literature

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